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French-U.S. Theory Institute for Physics with Exotic Nuclei

Some links between nuclear experiments and neutron star crust:

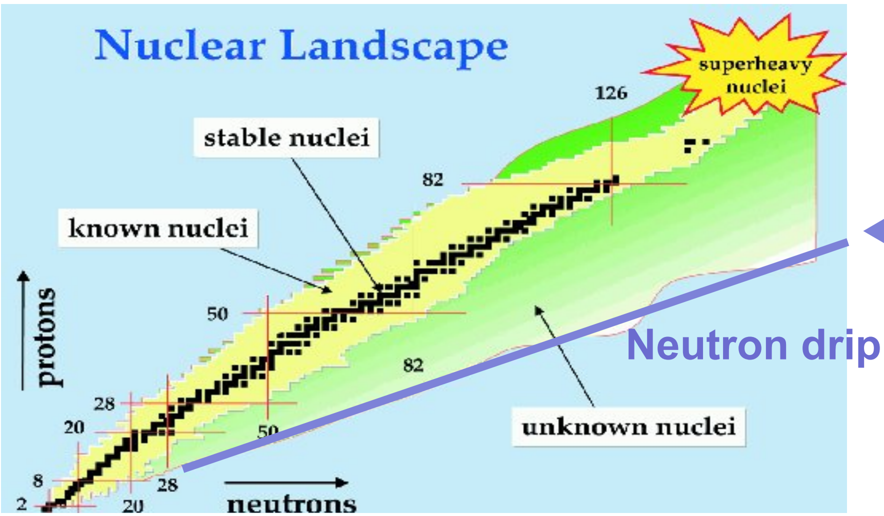
J. Margueron, IPN Lyon, France.

1- Superfluidity close to the neutron drip line and resonant states

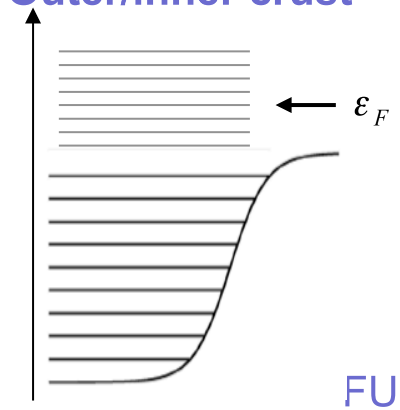
- Superfluidity at T=0: Suppression and persistence of pairing
- Re-entrance of pairing at finite temperature

2- Role of empirical coefficients (ρ_0, K_0, E_{sym}, L_{sym}, …)

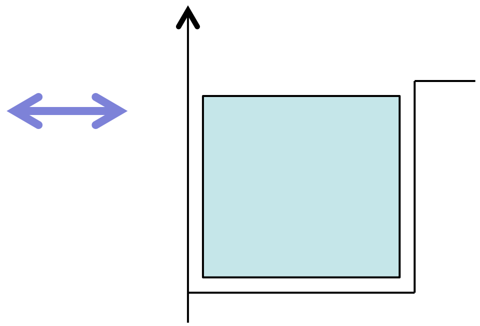
- Dilute clusters in the crust
- Impact on the nuclear equation of state



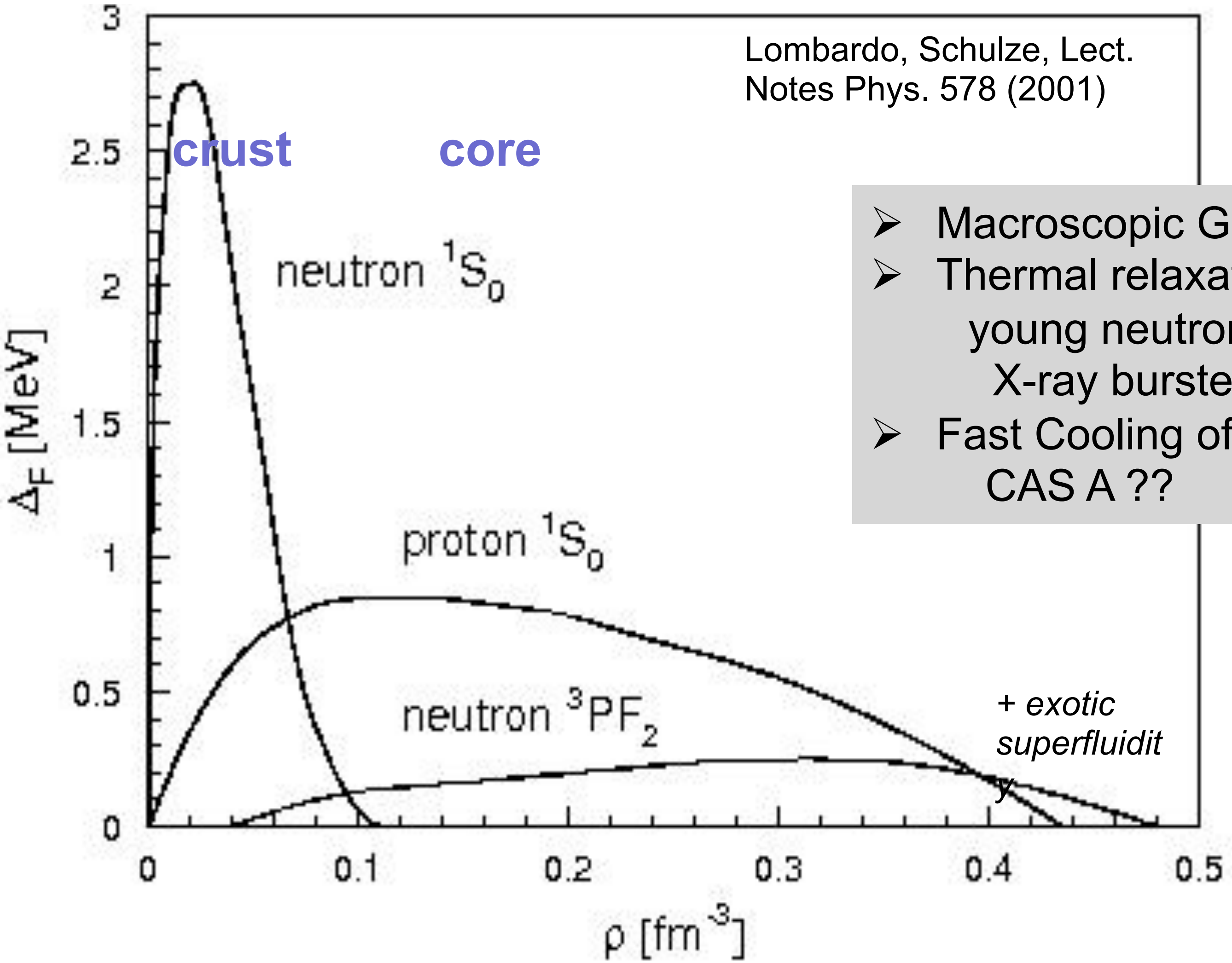
Transition Outer/inner crust



Equivalent piece of nuclear matter

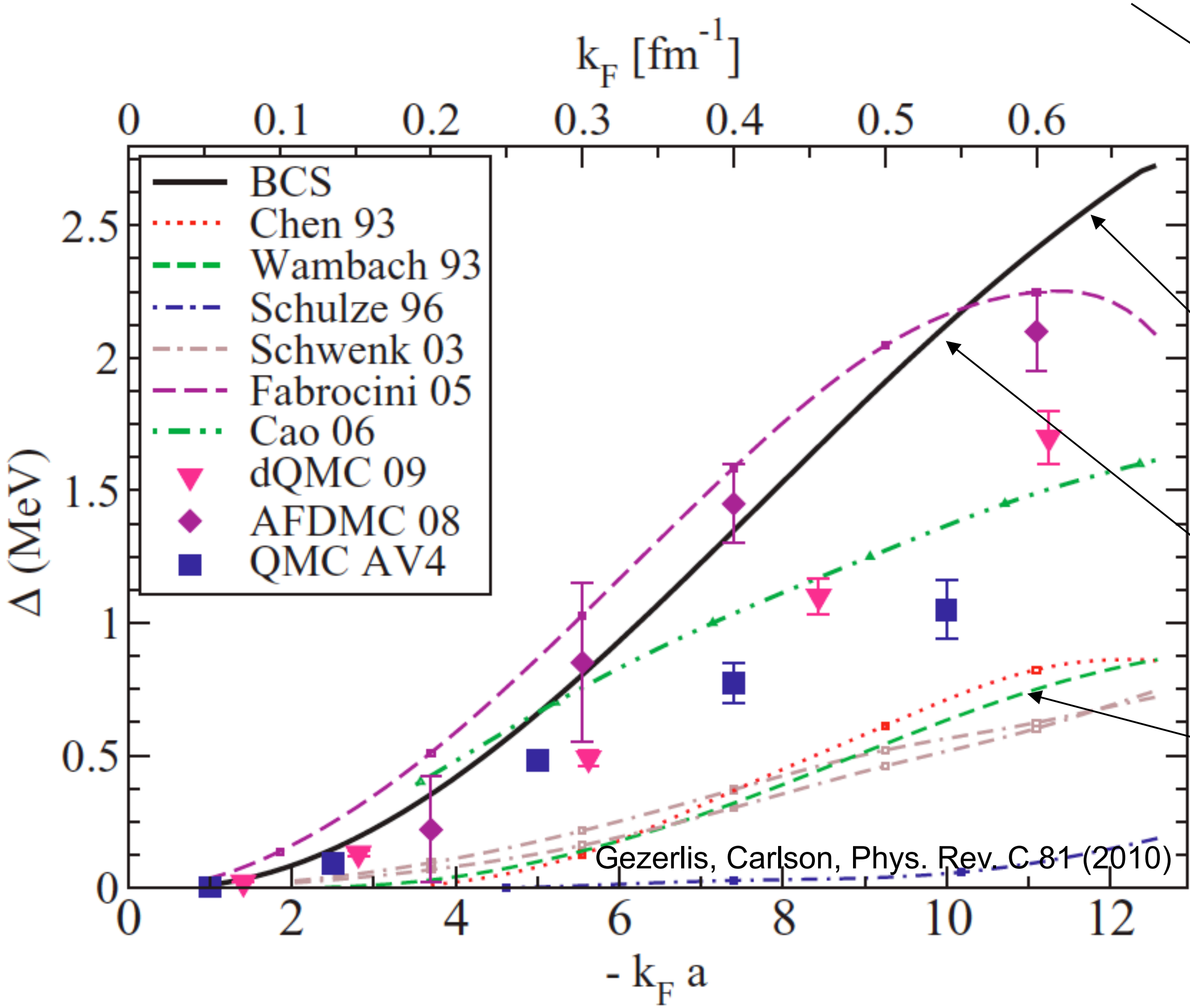


Superfluidity is present everywhere from the crust to the core



- Macroscopic Glitches
- Thermal relaxation of the crust
  - young neutron stars
    - X-ray bursters
- Fast Cooling of young neutron star ?
  - CAS A ??

$^1S_0$ pairing the "crust": large "theoretical errorbars"



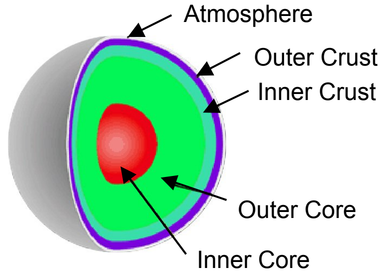
Needs for observational constraints

BCS

Strong (max at 3 MeV),

Weak (max at 1 MeV).

Probing NS crust through its thermal relaxation



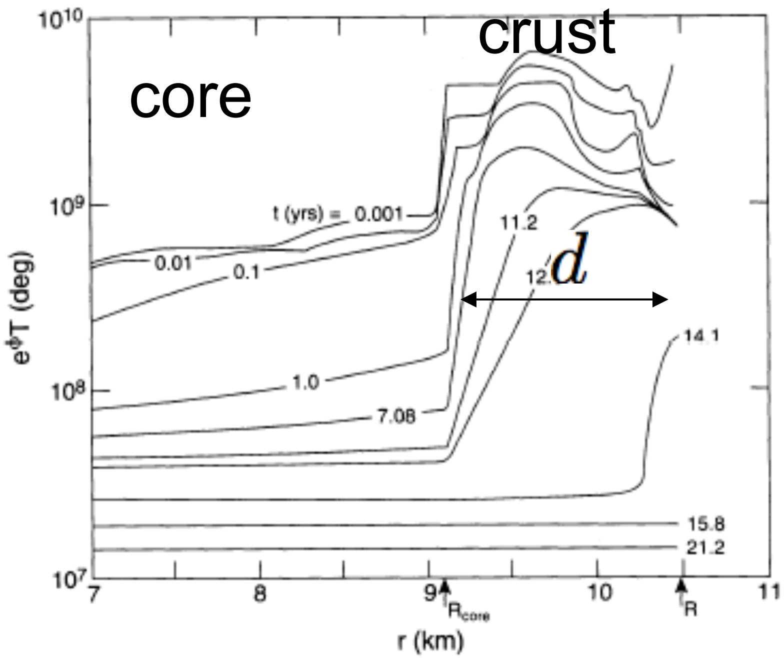
<u>Fast cooling of the core:</u>

➙ after ~1 year: Tcore << Tcrust ~0.5 MeV,

➙ next ~10-100 years: **thermalisation** of the crust:

$$\tau \propto \frac{d^2}{D}$$

with $$D = \frac{K}{\sum_i C_{v,i} \approx C_{v,n}}$$



Lattimer et al., APJ 425 (1994)

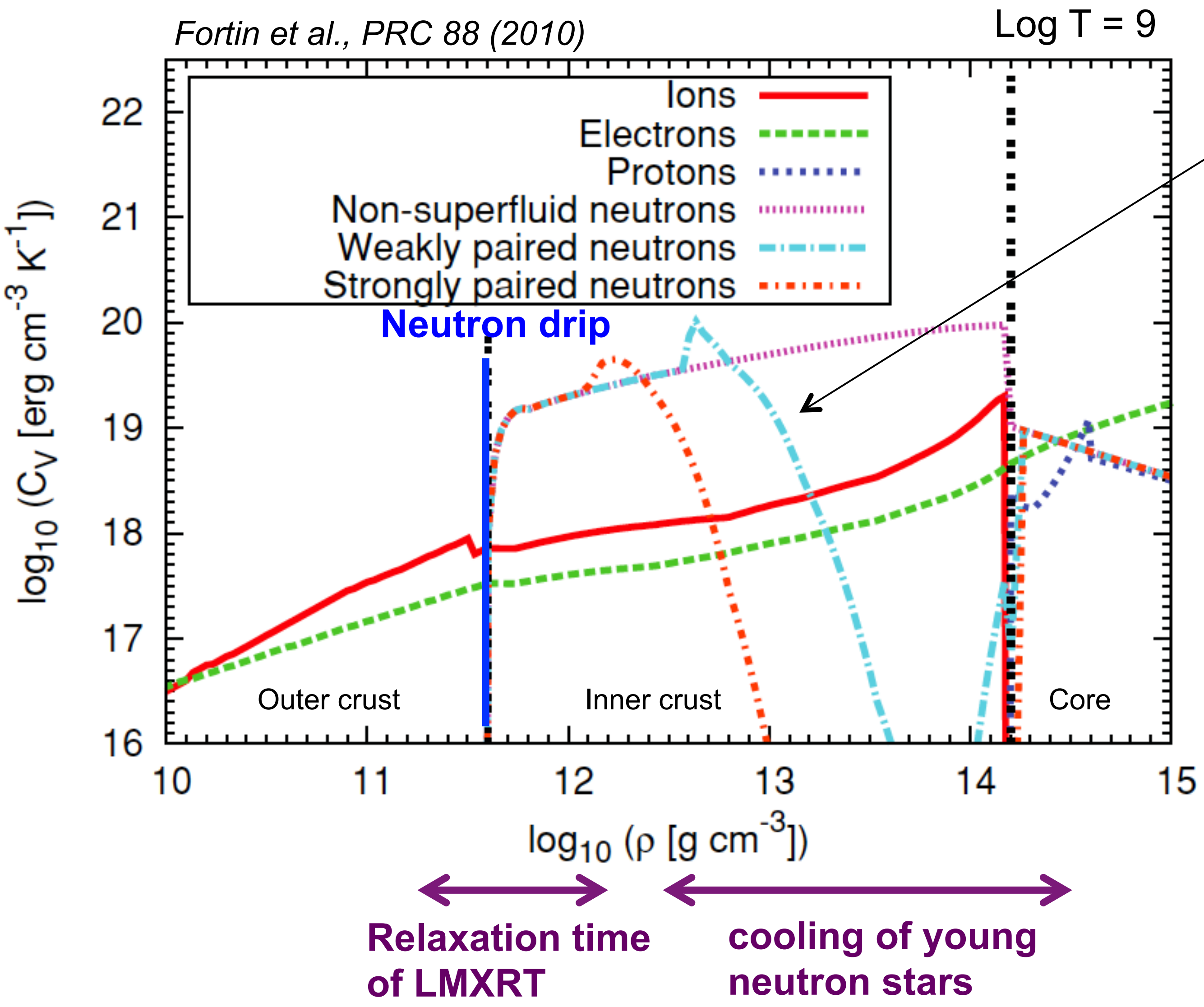
K, conductivity

$C_{v,n}$ neutron specific heat

depend on the cluster structure
In the neutron star crust

1- Superfluidity and cooling of neutron stars

Fortin et al., PRC 88 (2010)



Suppression of Cv in the superfluid phase

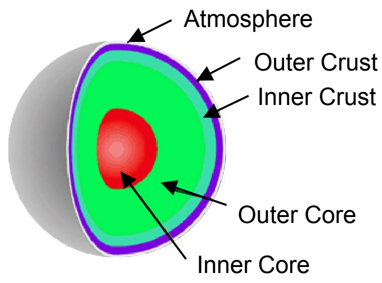
↓

Increase the diffusivity in the crust (D=K/Cv)

↓

Reduces the thermal relaxation time of the crust (τ=R^2/D)

Thermal relaxation of Neutron stars



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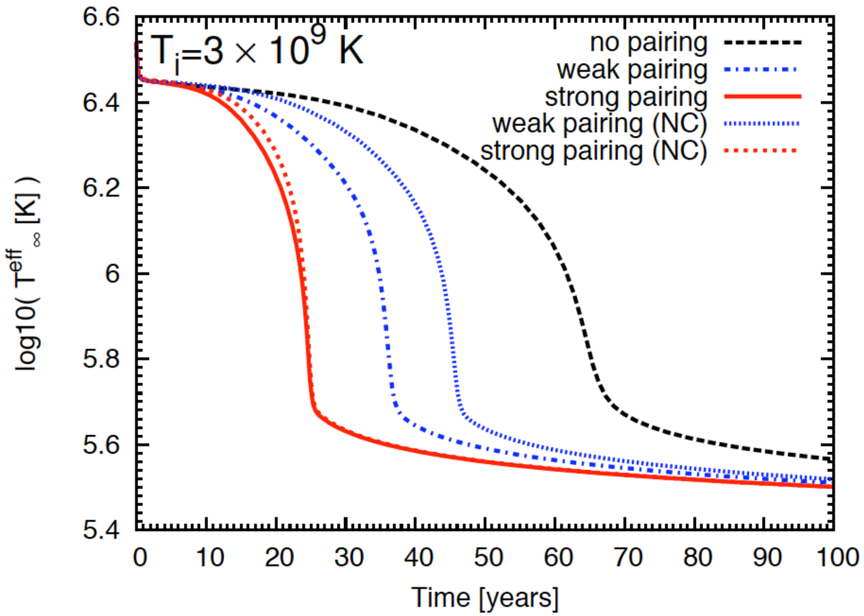


Fortin et al., PRC 88 (2010)

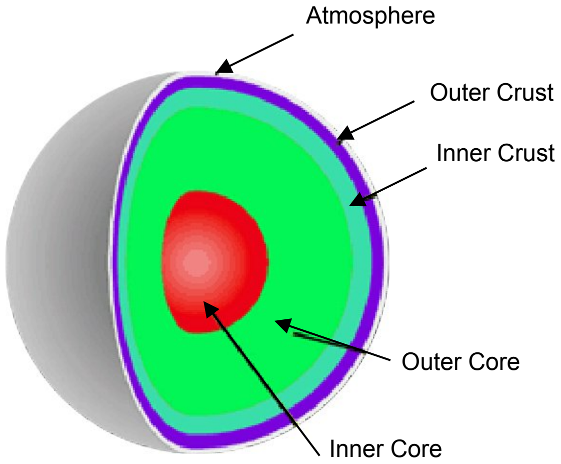
K, conductivity
$C_{v,n}$ neutron specific heat

} depend on the cluster structure
In the neutron star crust

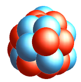
Effect of clusters larger for weak pairing.

Superfluidity at T=0

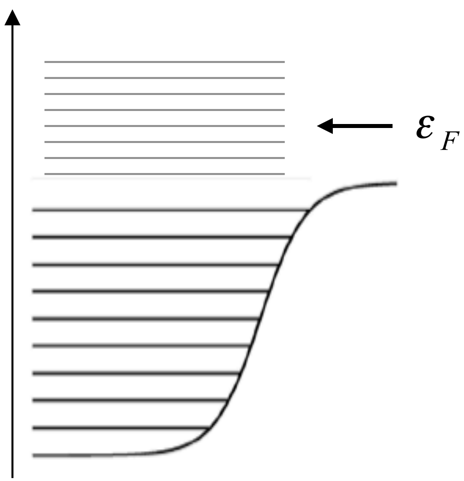
Neutron star



Finite nucleus



Transition outer / inner crust



Cold catalysed cells (Negele & Vautherin)

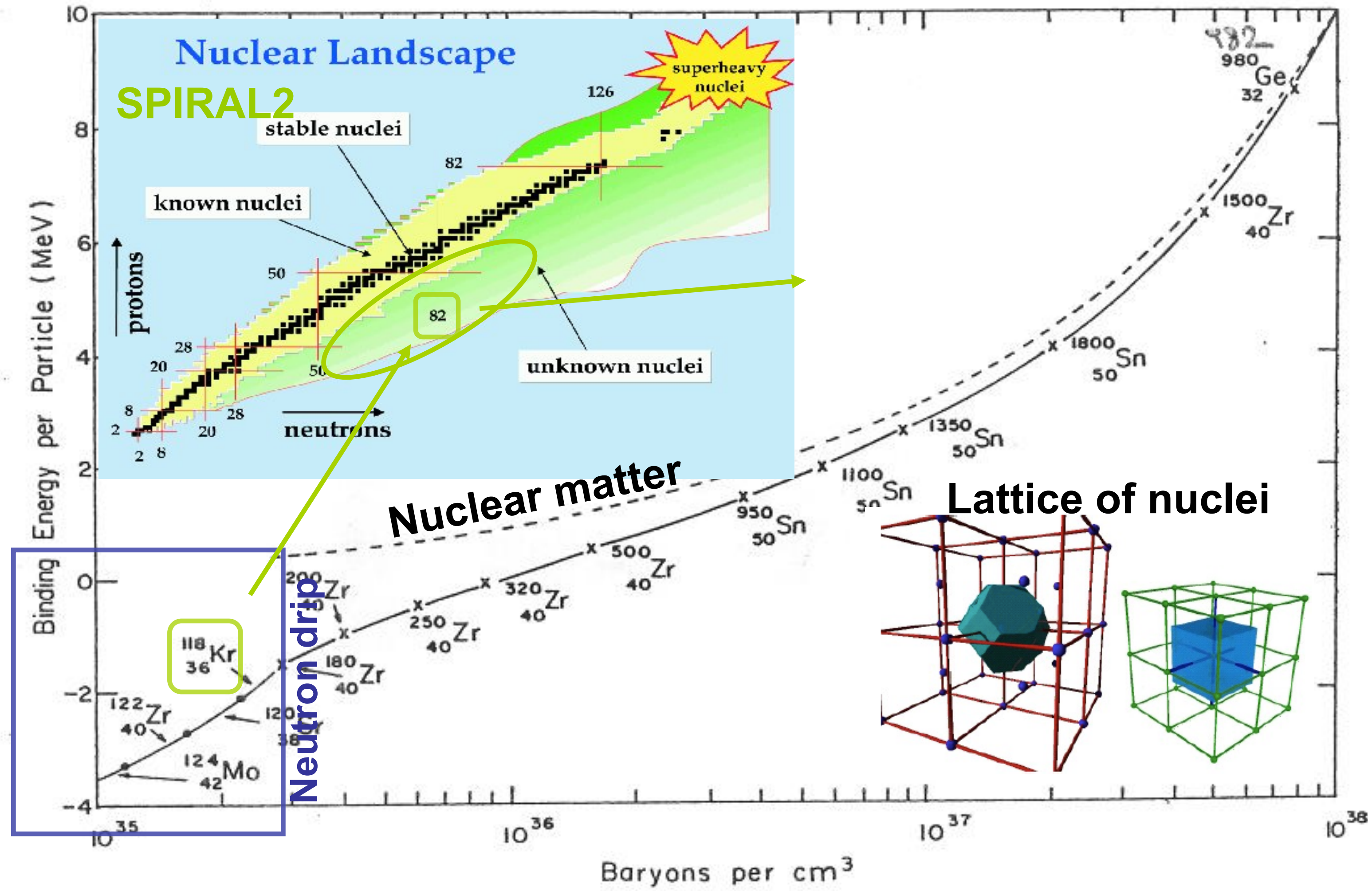
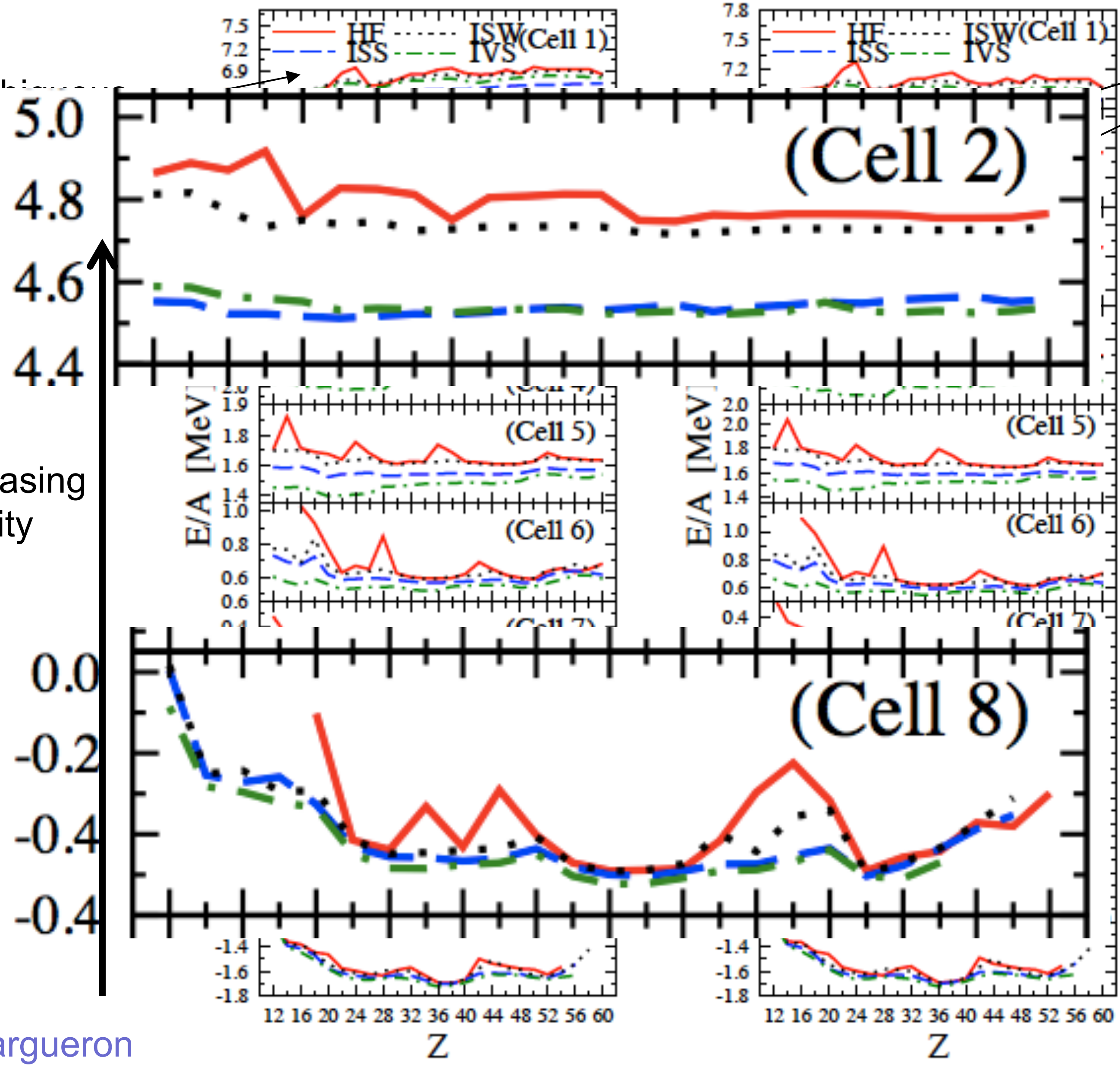


Fig. 2. Energy per particle versus baryon density.

Search for minimum energy

HFB **HFB + finite size corrections**



Am
abs
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Increasing density

Well defined absolute minimum

Smoothing of the Energy, Large influence of the **superfluid neutrons**.

Large **nuclear shell structure**

Final results

N_{cell}	N				Z				Z_{corr}			
	HF	ISW	ISS	IVS	HF	ISW	ISS	IVS	HF	ISW	ISS	IVS
2									40	40	22	42
3	318	514	442	554	16	24	20	24	54	40	28	38
4	476	534	382	570	28	28	20	28	40	40	40	28
5	752	320	328	344	46	20	20	20	46	48	44	20
6	454	428	374	346	50	48	36	34	50	50	50	34
7	316	344	324	240	50	50	50	36	50	50	50	50
8	174	220	186	174	36	50	38	36	36	50	38	36
9	120	112	128	116	38	36	38	36	38	36	38	36
10	94	82	90	82	38	36	38	36	36	36	36	36

F. Grill, J.M., N. Sandulescu, Phys. Rev C (2011)

Conclusions:

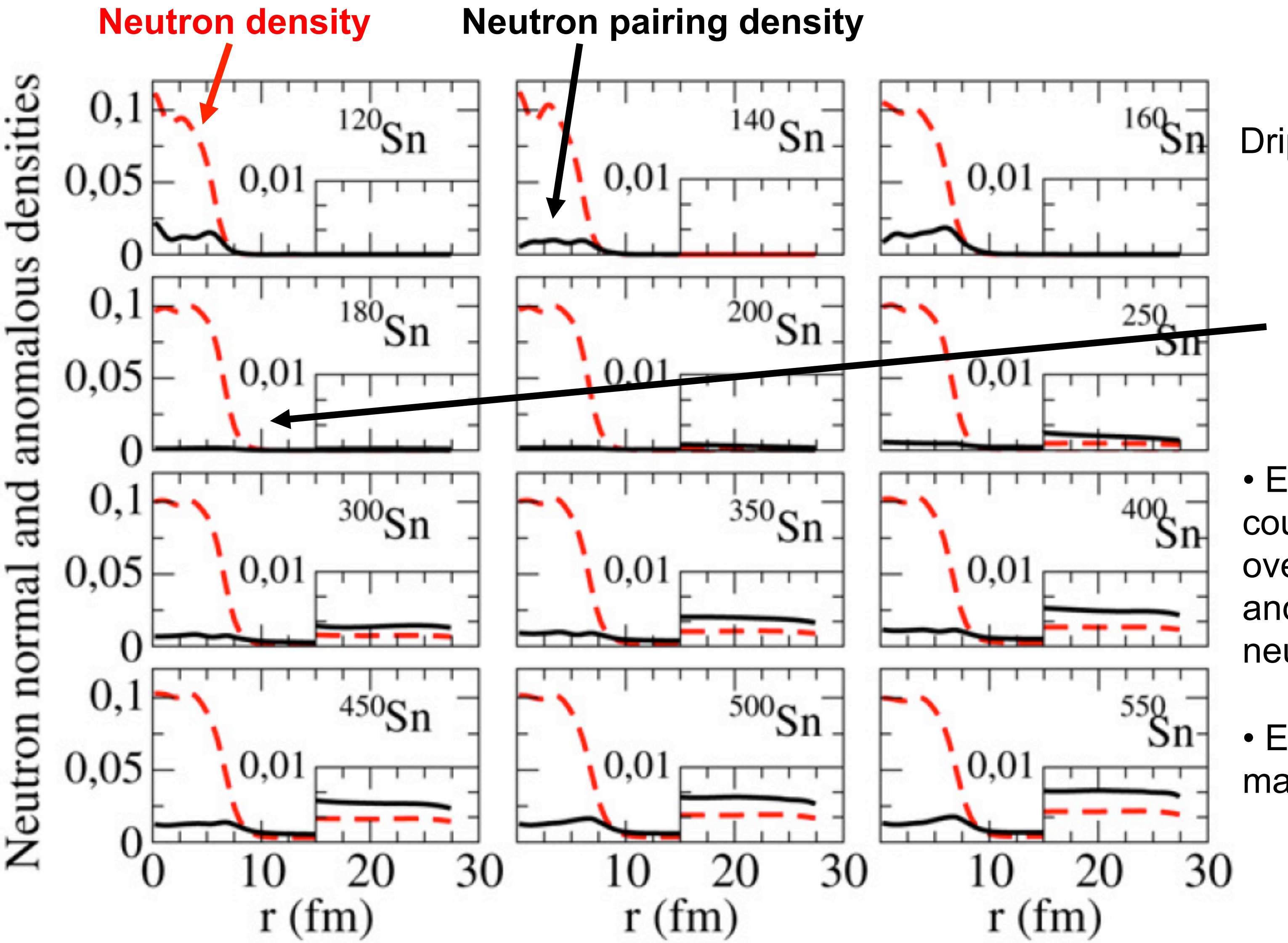
Reduction of the stabilization induced by shell effect (compared to Negele-Vautherin).
Delicate energy balance between the cells.
Pairing smoothes the energy surface & can change significantly Z for some cells.

Outlooks:

Effects of these new results on the cooling (conductivities, specific heat), glitches, …

The role of resonant states?

Grasso et al., Nucl. Phys. A 807 (2008)

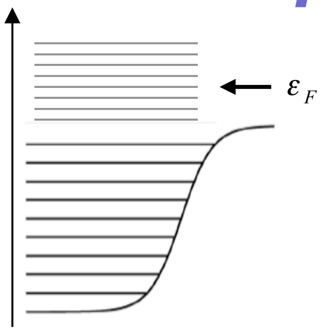


Drip nucleus: ^{176}Sn

- Effect of the coupling between overflowing neutrons and trapped neutrons?
- Effects of magicity?

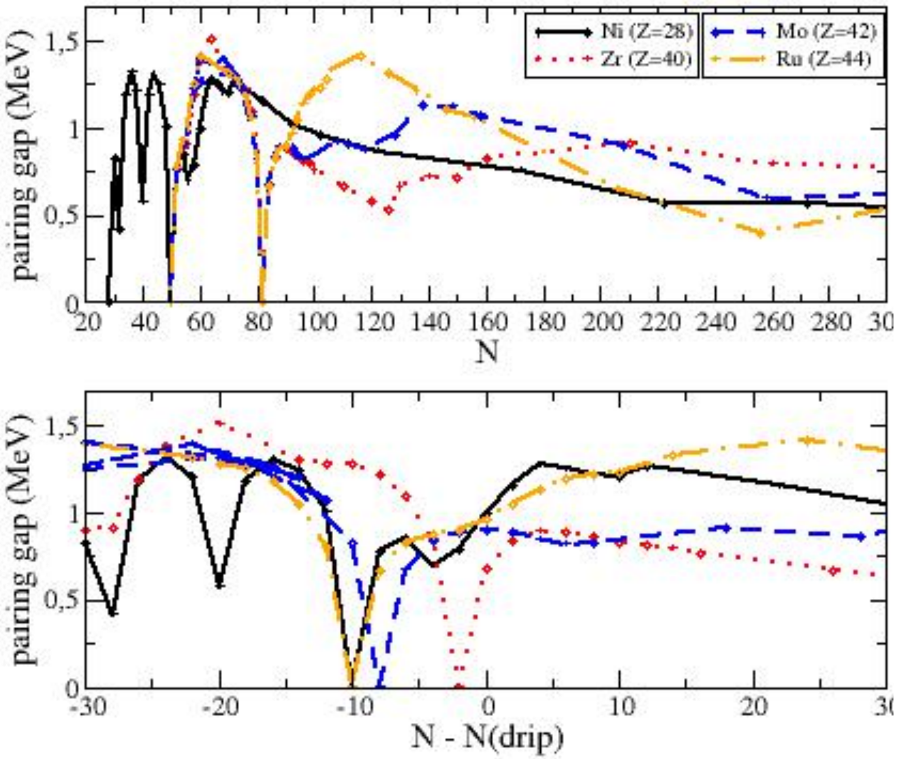
A systematic study based on 8 isotopes with 28<Z<50

Nucleus	Z	N_{drip}	group	N_{res}
Ni	28	60	$\mathcal{A}_1$	3.0
Kr	36	82	$\mathcal{A}_2$	0.0
Sr	38	82	$\mathcal{A}_2$	0.0
Zr	40	84	$\mathcal{A}_1$	2.2
Mo	42	90	$\mathcal{A}_1$	8.0
Ru	44	92	$\mathcal{A}_1$	3.0
Sn	50	126	$\mathcal{A}_2$	0.0
Te	52	126	$\mathcal{A}_2$	0.0

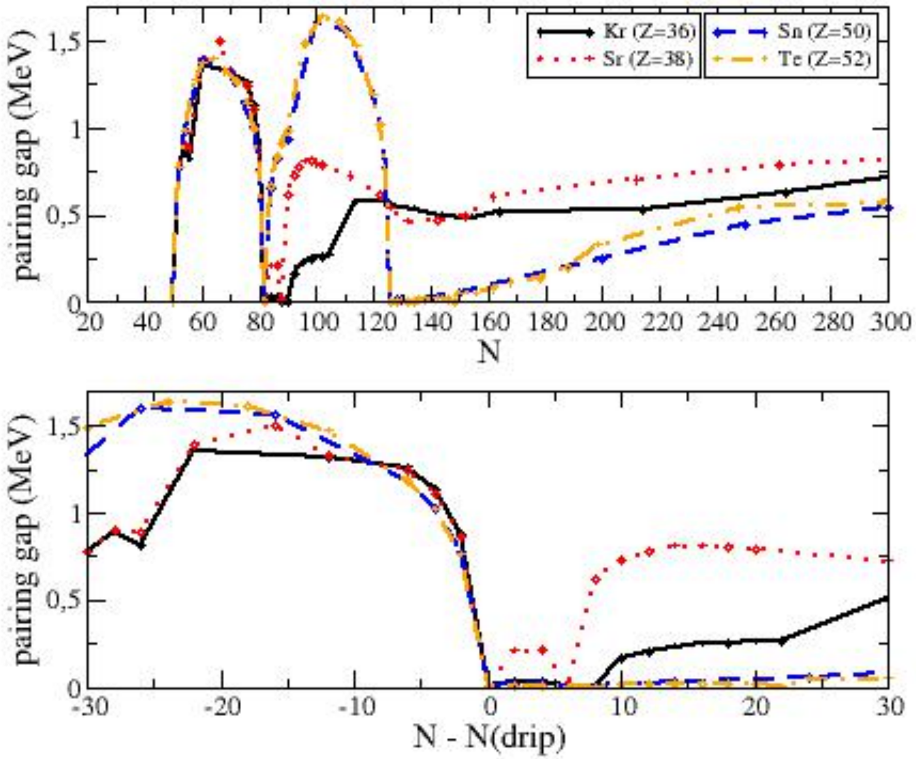


Pairing is suppressed in the absence of occupied resonant states at the drip-line.

group $\mathcal{A}_1$



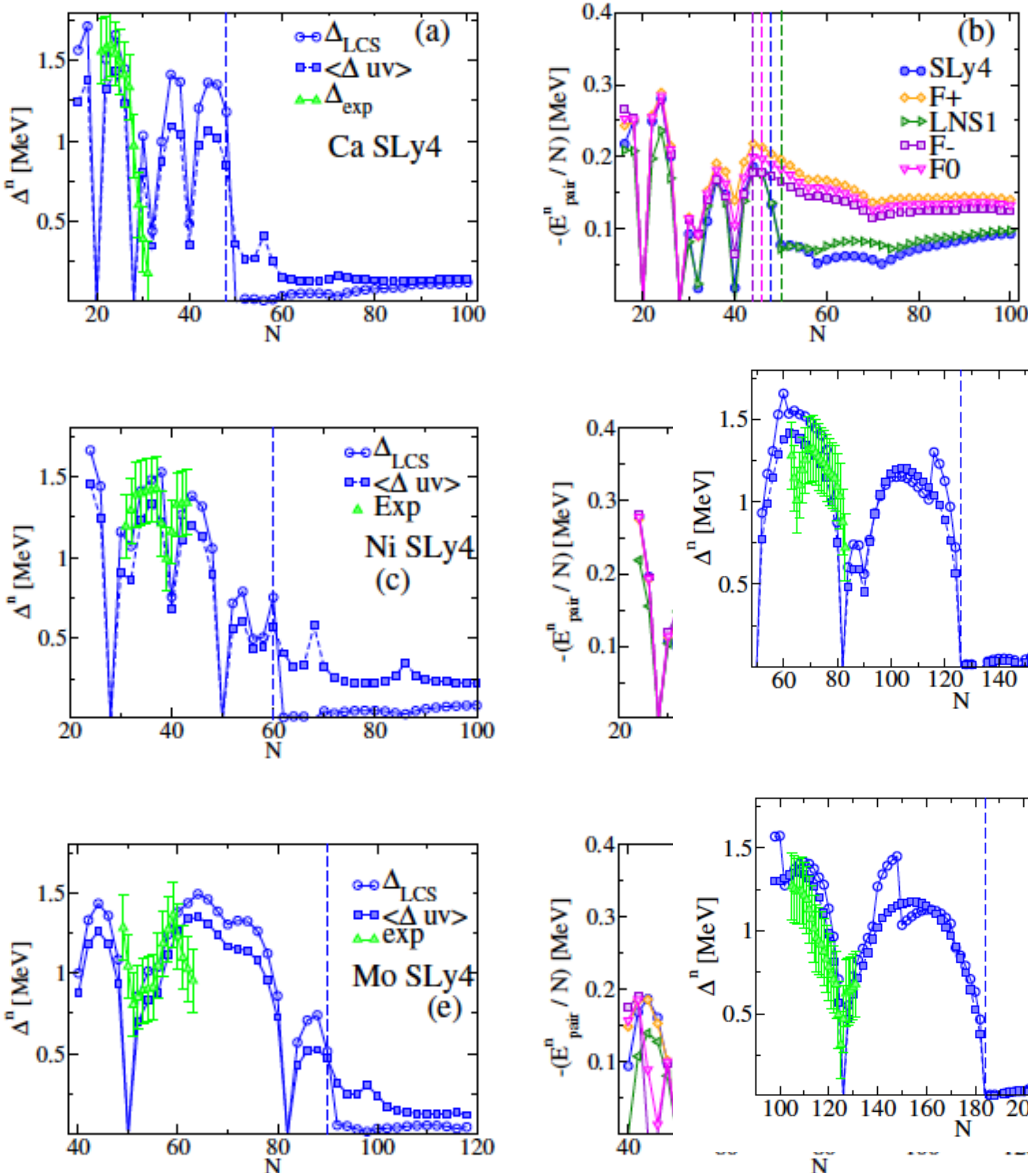
group $\mathcal{A}_2$



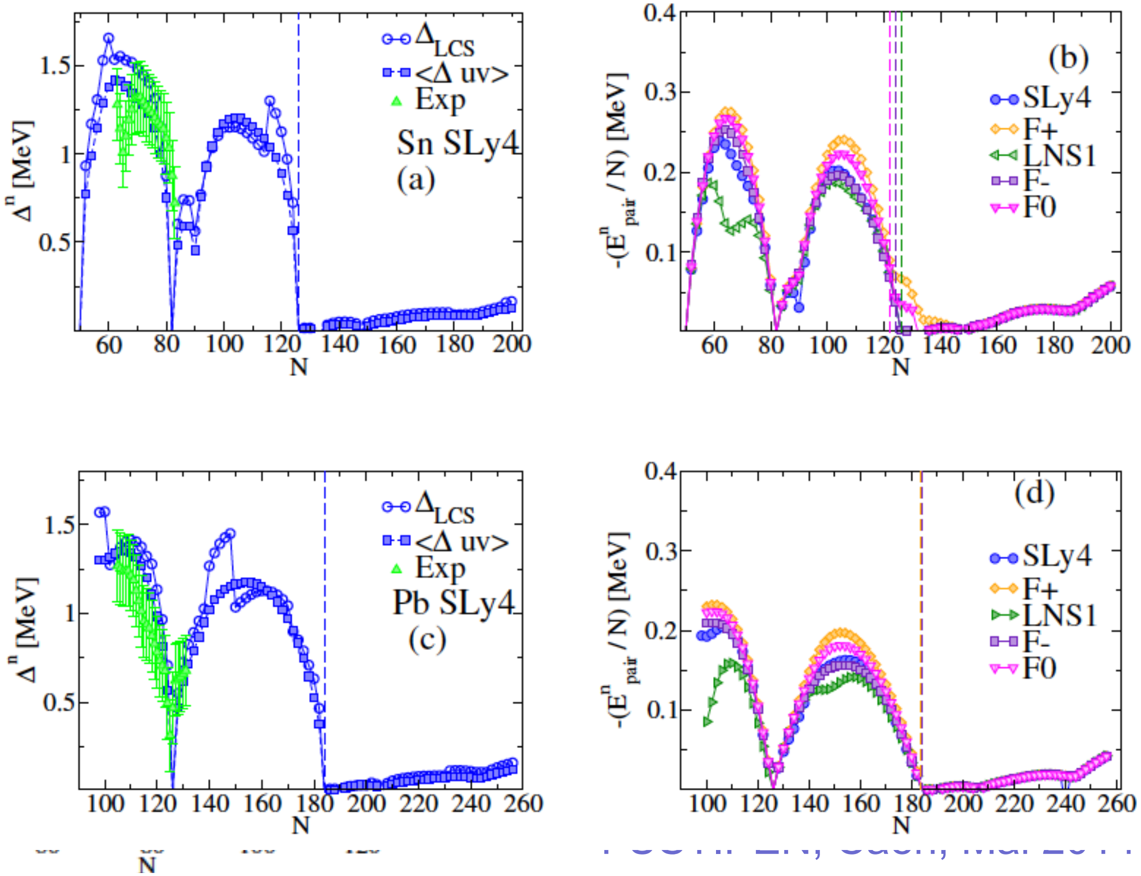
Weak dependence on the model

Pastore et al., PRC 2013

Group A_1



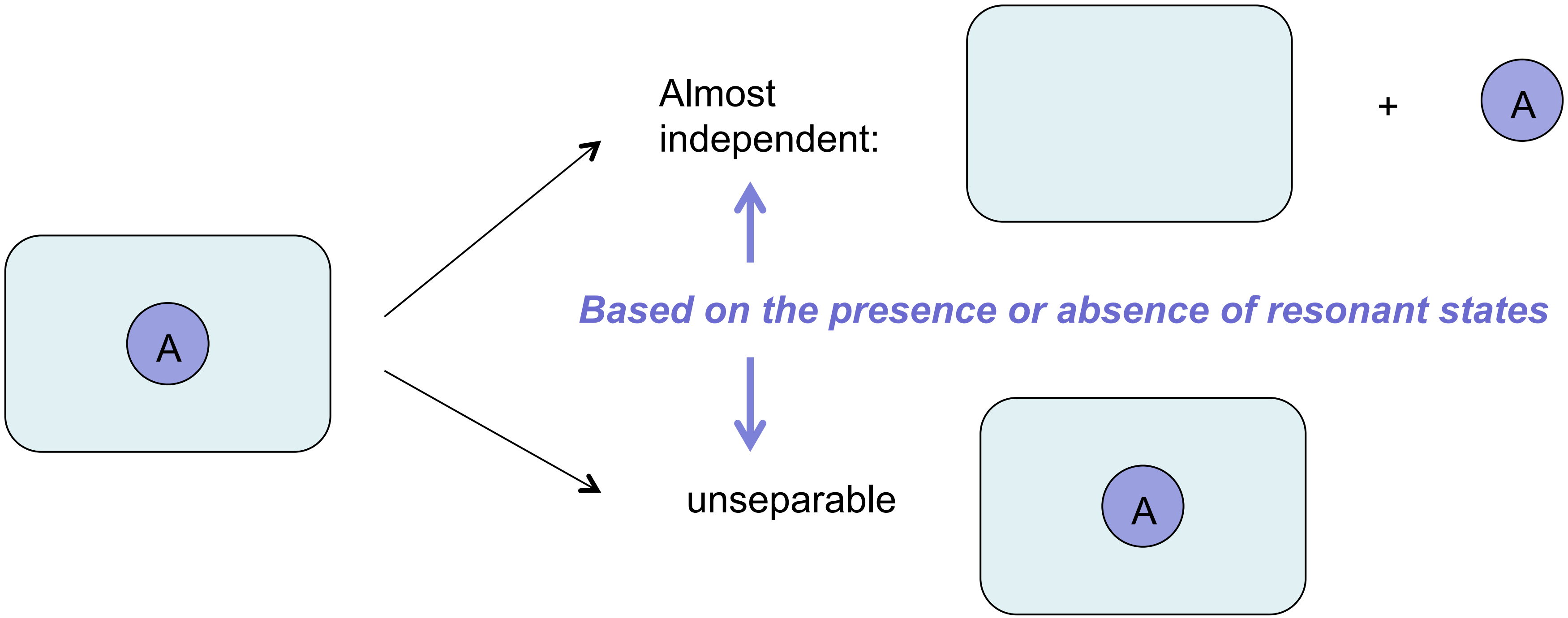
Group A_2



A simple picture beyond the drip?

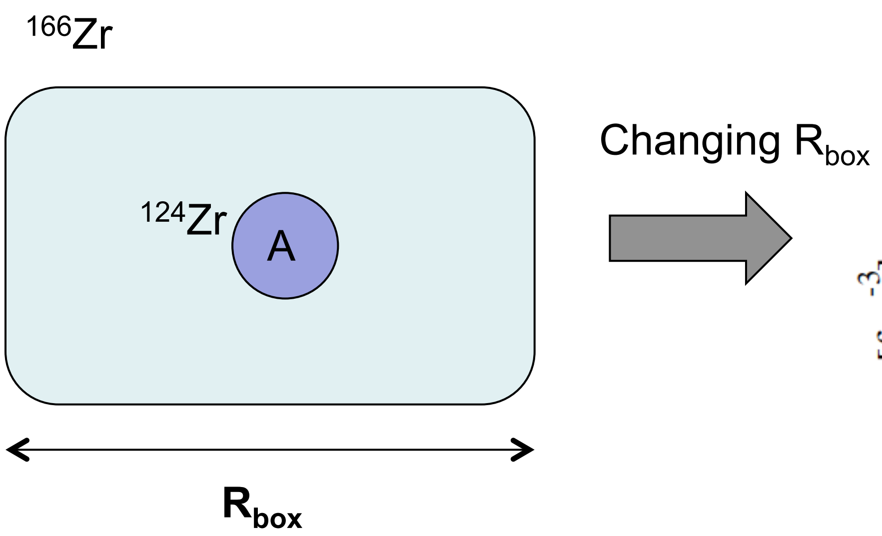
What is the interplay between the gas and the nuclei ?

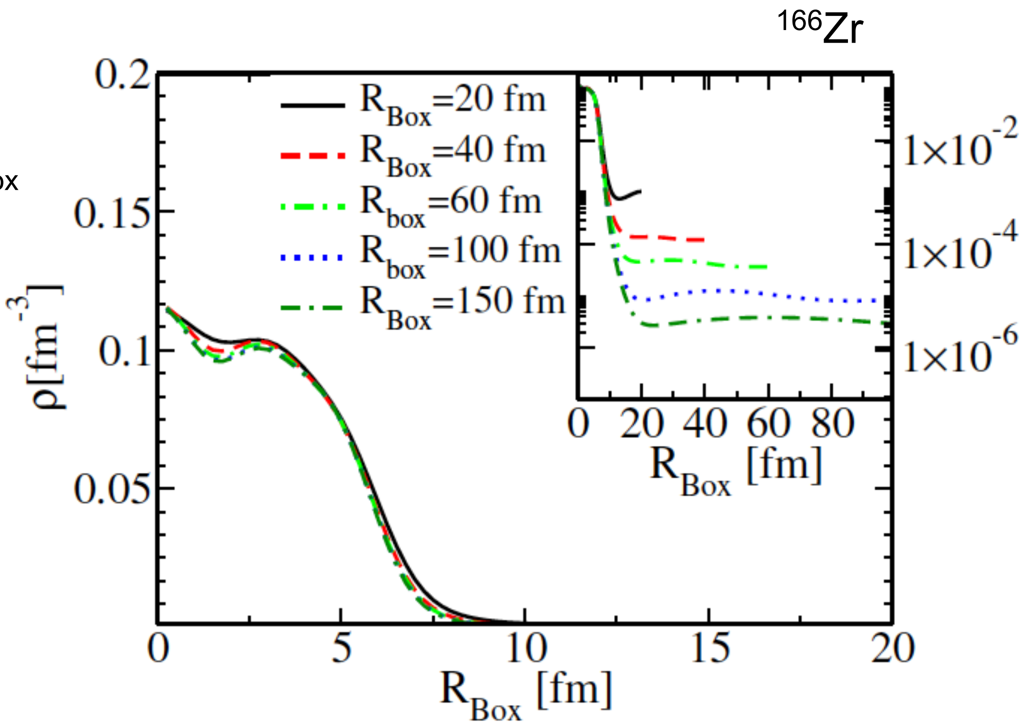
Depending on the shell structure of drip nucleus:



The limit of very dilute clusters

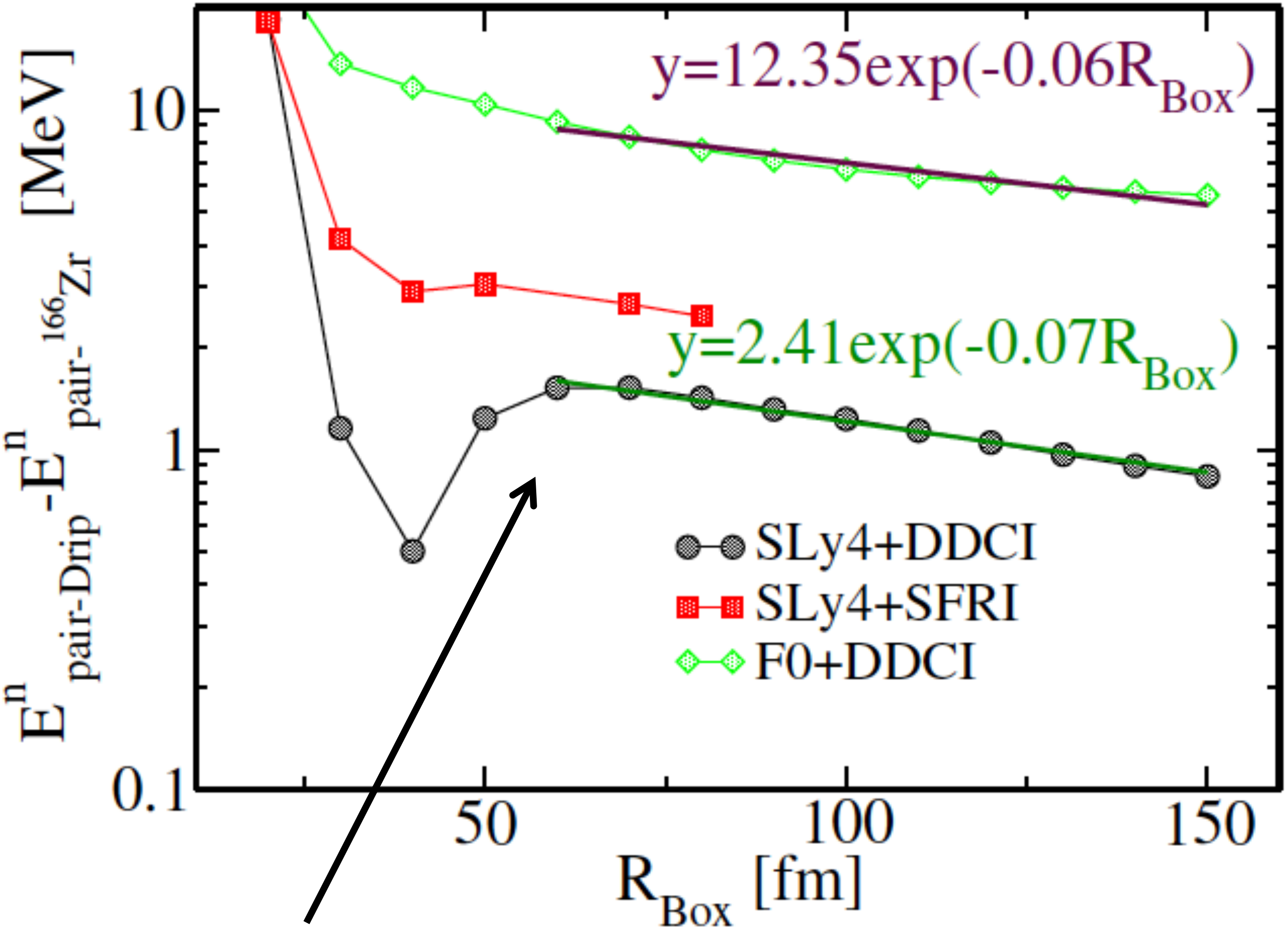
Interaction of a shallow gas with a nucleus





At which density ρ_{gas} the gas can be neglected?

Decreasing the gas density (by increasing the volume)

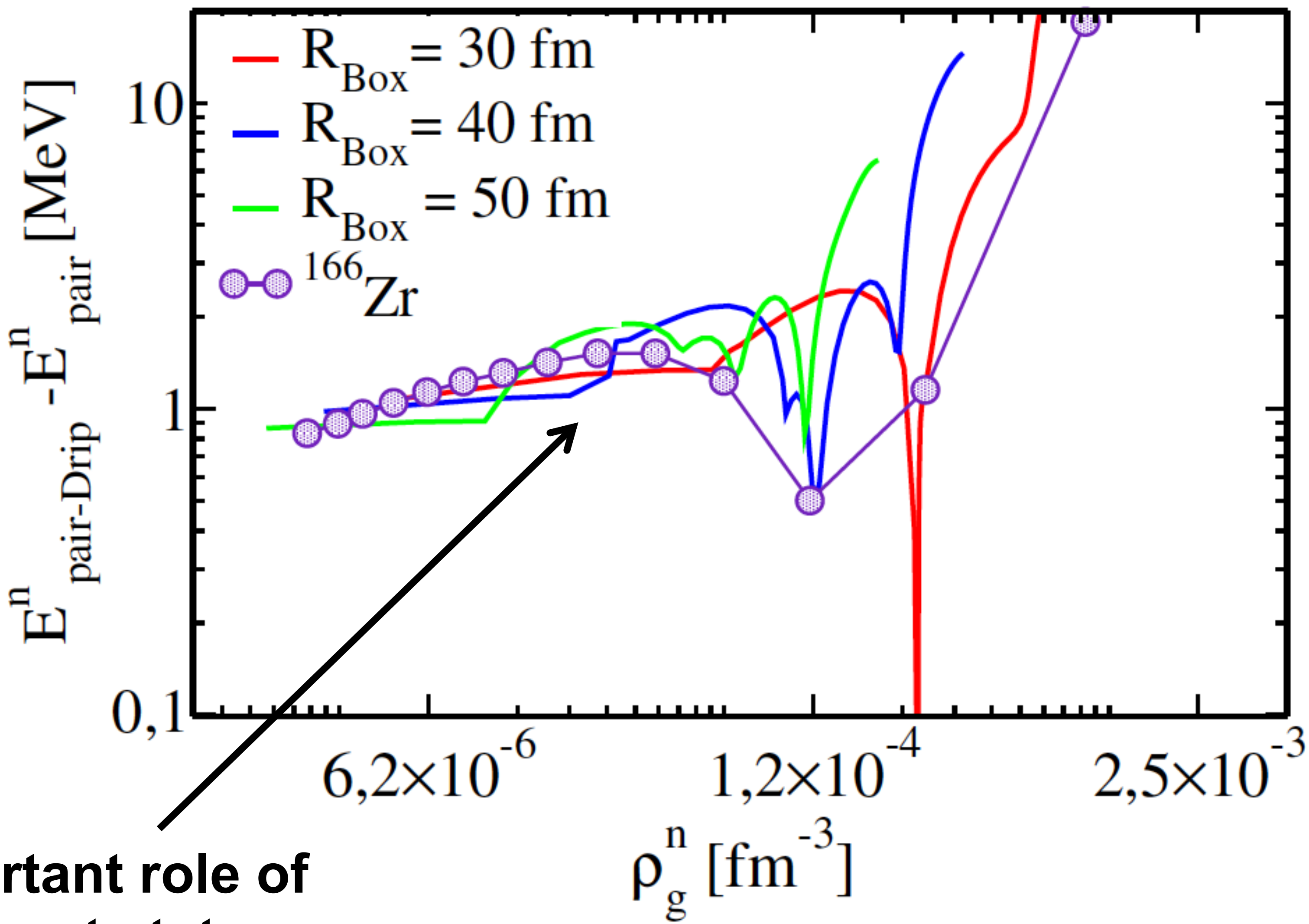


Important role of resonant states

Pastore et al., PRC 2013

Decreasing the gas density (by decreasing N)

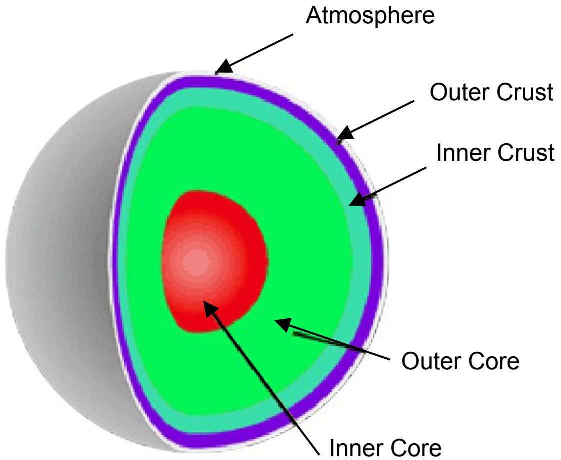
Fix Rbox, and decrease the total number of neutrons



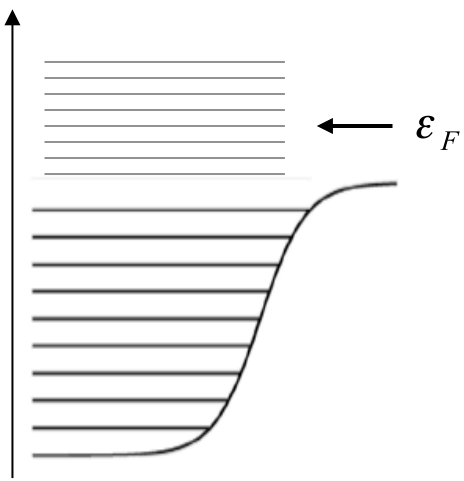
Important role of resonant states

Pastore et al., PRC 2013

Finite temperature in non-uniform matter



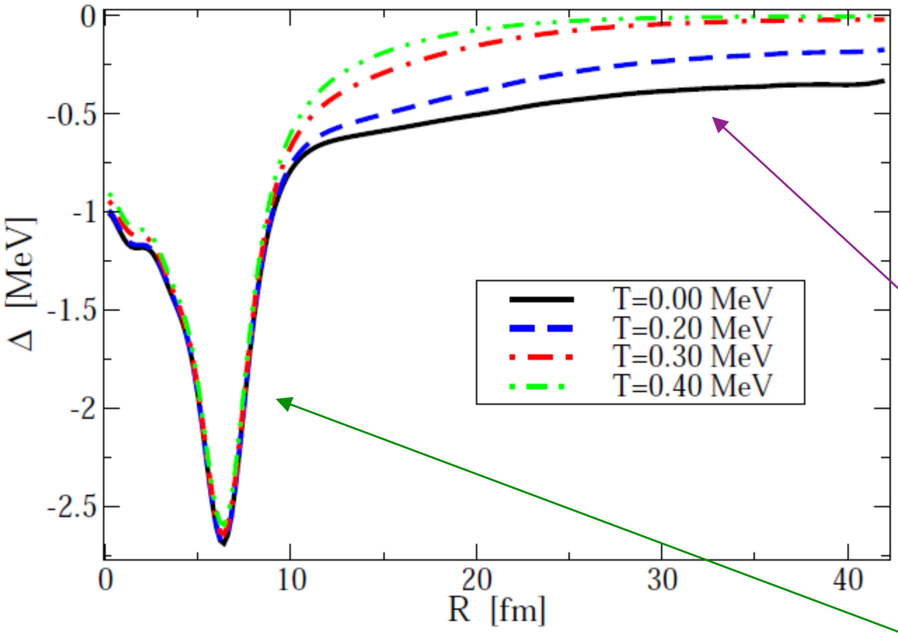
Transition outer / inner crust



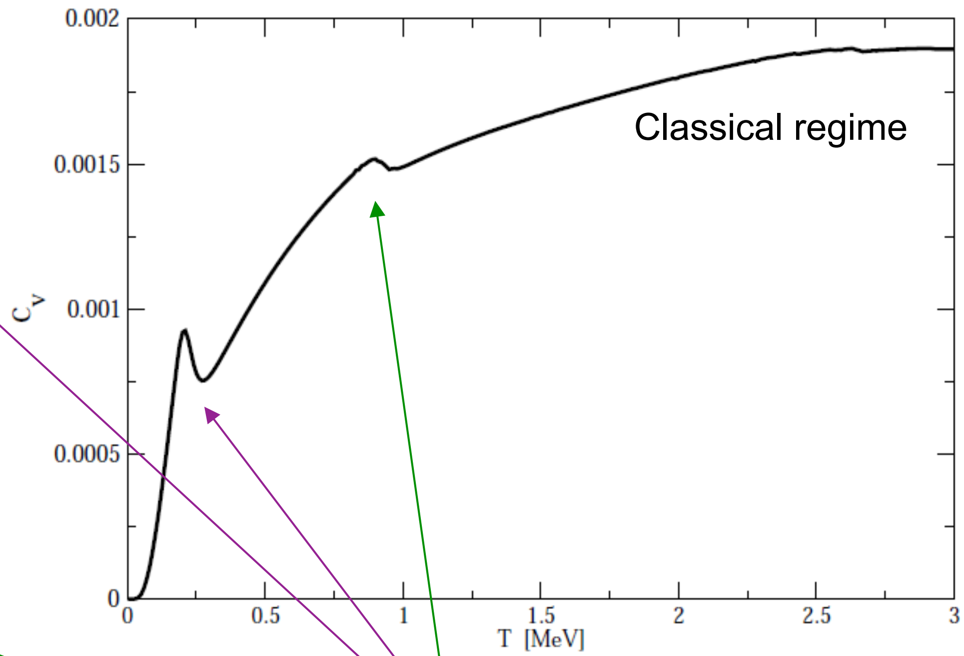
Neutrons specific heat in ^{500}Zr

N=460, Z=40

Pairing field profile
at various temperatures:



Neutron specific heat:



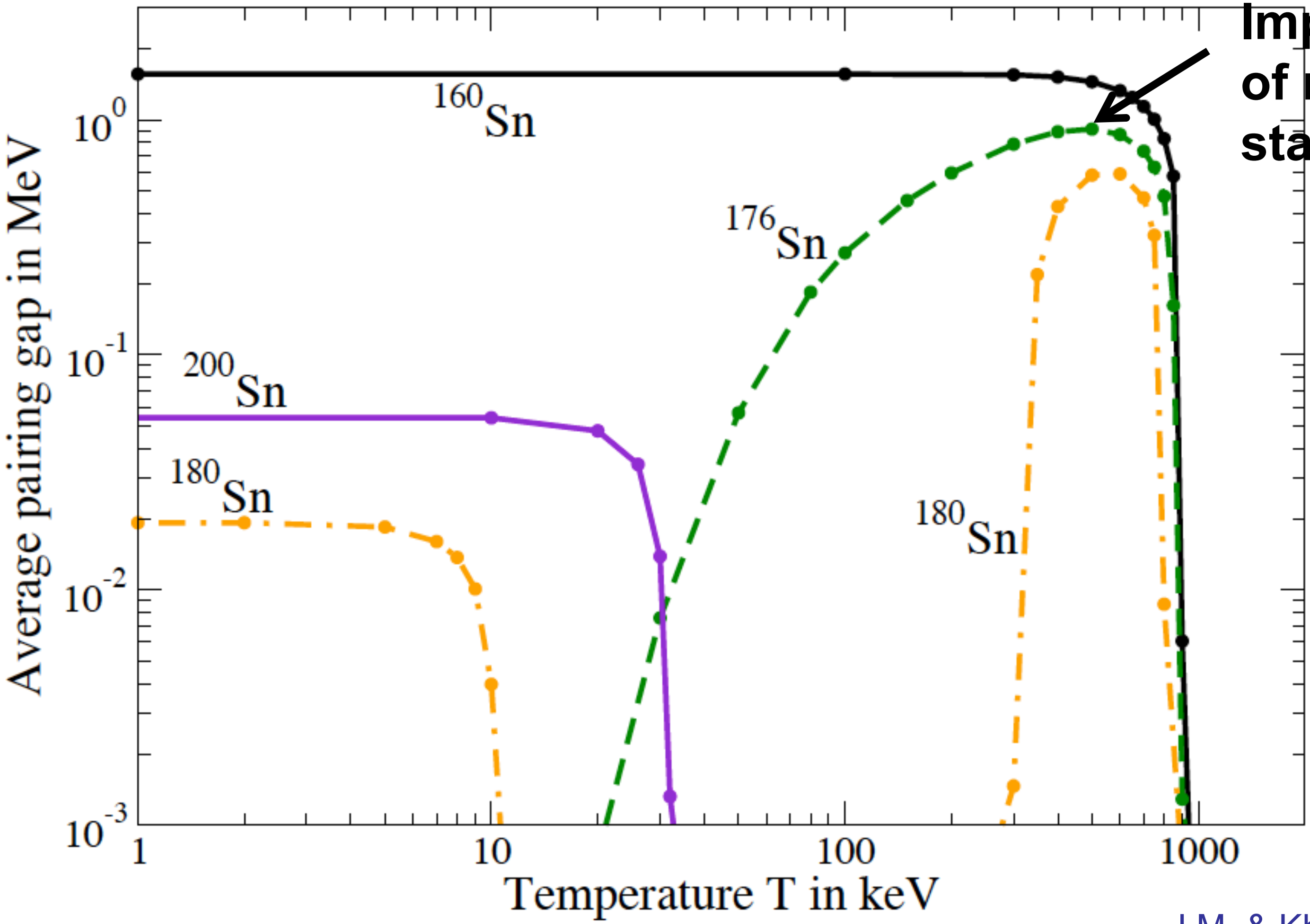
Disappearance of superfluidity

in the neutron gas

in the cluster

Fortin et al., PRC 88 065804 (2010)

Pairing reentrance in Sn at the drip



Important role of resonant states

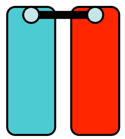
J.M. & Khan, PRC 2012

Temperature populates excited states:
1- kinetic energy cost induces a quenching of pairing,
2- in some cases, pairing occurs among thermally occupied excited states.

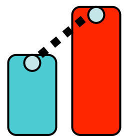
Pairing reentrance phenomenon

Superfluidity is destroyed by increasing the temperature…
But a bit of temperature sometimes helps in restoring superfluidity !

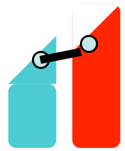
Pairing reentrance in asymmetric systems:



Pairing in symmetric systems



Asymmetry detroys pairing



Temperature in asymmetric systems restore superfluidity

In nuclear matter: pairing in the T=0 (deuteron) channel

Sedrakian, Alm, Lombardo, PRC 55, R582 (1997)

In spin-asymmetric cold atom gas

Castorina, Grasso, Oertel, Urban, Zappala, PRA 72, 025601 (2005)
Chien, Chen, He, Levin, PRL 97, 090402 (2006)

In higly polarized Liquid ^{3}He, ^{4}He

Frossati, Bedell, Wiegers, Vermeulen, PRL 57 (1986)

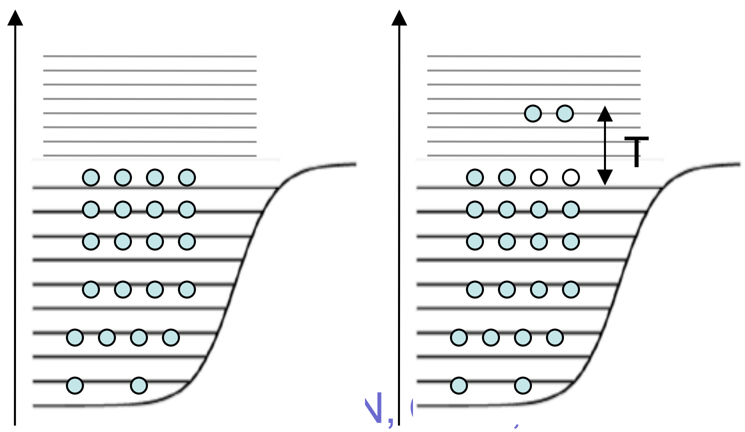
Pairing in heated rotating nuclei

Dean, Langanke, Nam, and Nazarewicz, PRL105, 212504 (2010).

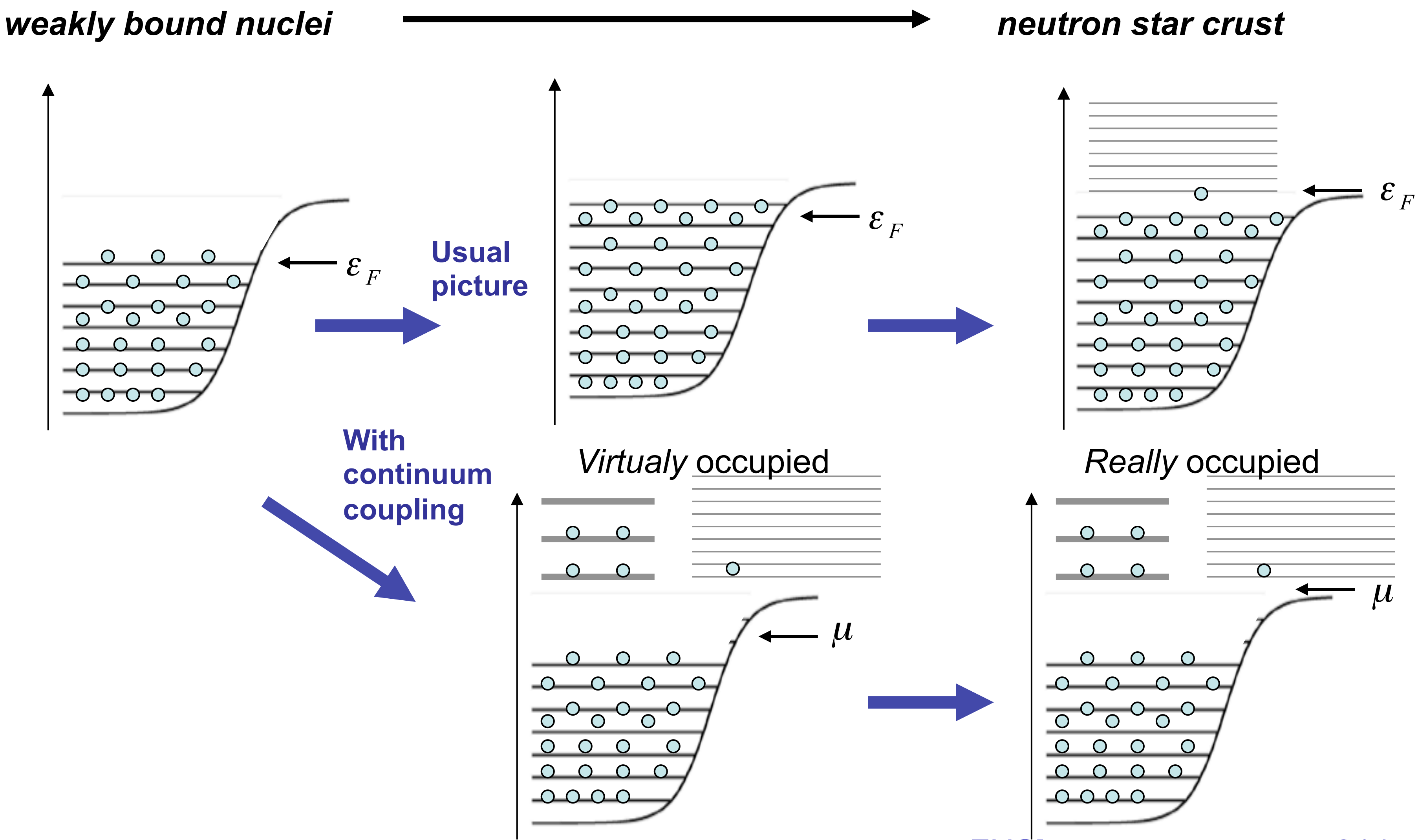
Pairing reentrance in finite systems:

In magic nuclei, the presence of low-energy resonances, populated at low temperature, can help superfluidity to appear.

J.M., Khan, PRC 2012



Towards a better understanding of the shell effect around the neutron drip



Conclusions:

- The transition between the **outer / inner crust** offers a fascinating playground to apply and test pairing theories.

- Since two superfluids overlap (gas+nucleus), surprising features occurs, mostly due to the **resonant states**.

These non-trivial features of superfluidity are interesting for:

- Neutron stars: Models for the crust including pairing shall be revised taking into account finite temperature in non-uniform nuclear clusters.
- Could resonant states be responsible for the **entrainment** phenomenon?
- For **nuclear experiments**: role of continuum coupling going towards the drip line for 20<Z<40.

2- The role of empirical coefficients in compact star physics

The binding energy can be expressed as:

$$E(x,\delta) = E(x) + \delta^2 J(x) \quad \text{with} \quad \begin{array}{l} x = (n-n_0)/3n_0 \\ \delta = (N-Z)/A \end{array}$$

The density dependence of the EoS can be expressed in term of the parameters E_0, K_0, Q_0:

$$E(x) = E_0 + K_0 x^2 + Q_0 x^3 + \ldots$$

The isospin dependence is mostly quadratic and is expressed as

$$J(x) = J_{sym} + L_{sym} x + K_{sym} x^2 + \ldots$$

→ Links between experiments and these parameters.

GS density of dilute clusters in the crust of neutron stars

J. Margueron

FUSTIPEN, Caen, Mai 2014

A simple model for the GS density

4 Variables: (A,Z) cluster, (ρ, δ) gas

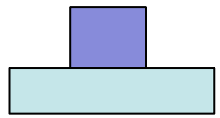
Wood-Saxon density profiles → 3 parameters (ρ_0, R, a)

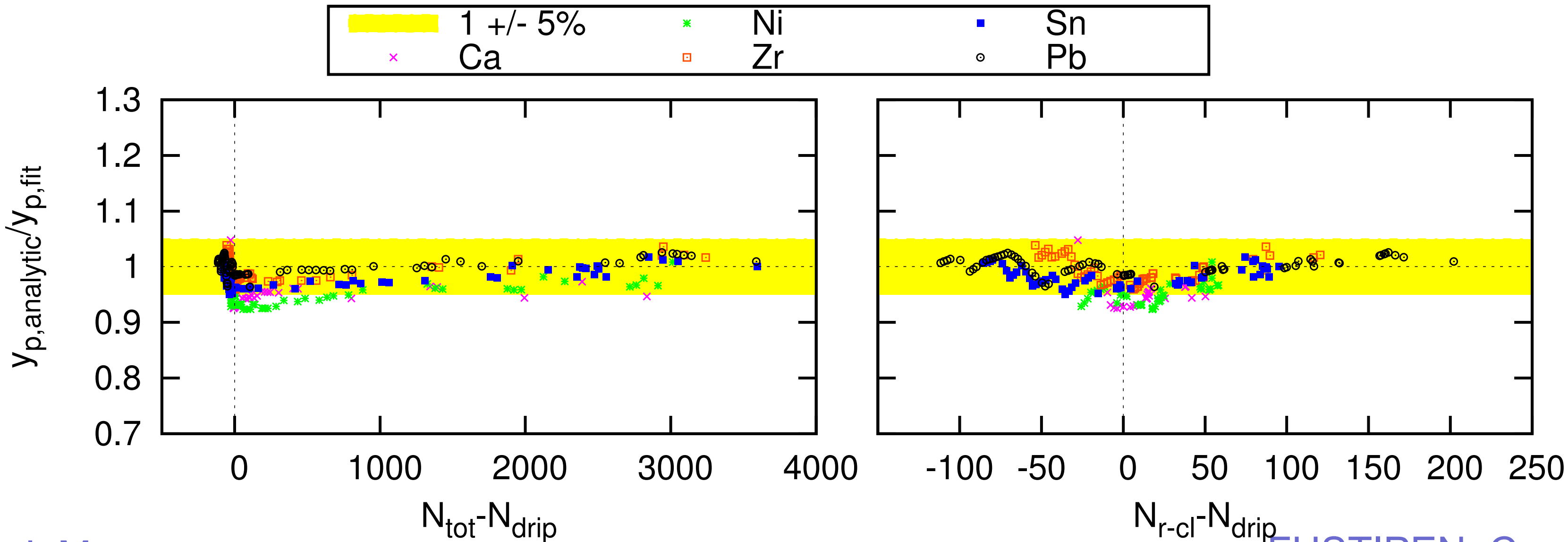
Hypothesis 1: the bulk asymmetry is not N-Z/(N+Z) but takes into account skin effects.

$$\delta_{e-cl} = \frac{I_{e-cl} + \frac{3a_C}{8Q}\frac{(Z_{e-cl})^2}{(A_{e-cl})^{5/3}}}{1 + \frac{9J_0}{4Q}\frac{1}{(A_{e-cl})^{1/3}}}$$

From Myers & Swiatecki LDM

$$\delta_{r-cl} = \left(1 - \frac{\rho_{gas}}{\rho_0(\delta_{r-cl})}\right)\delta_{e-cl} + \frac{\rho_{gas}}{\rho_0(\delta_{r-cl})}\delta_{gas}$$

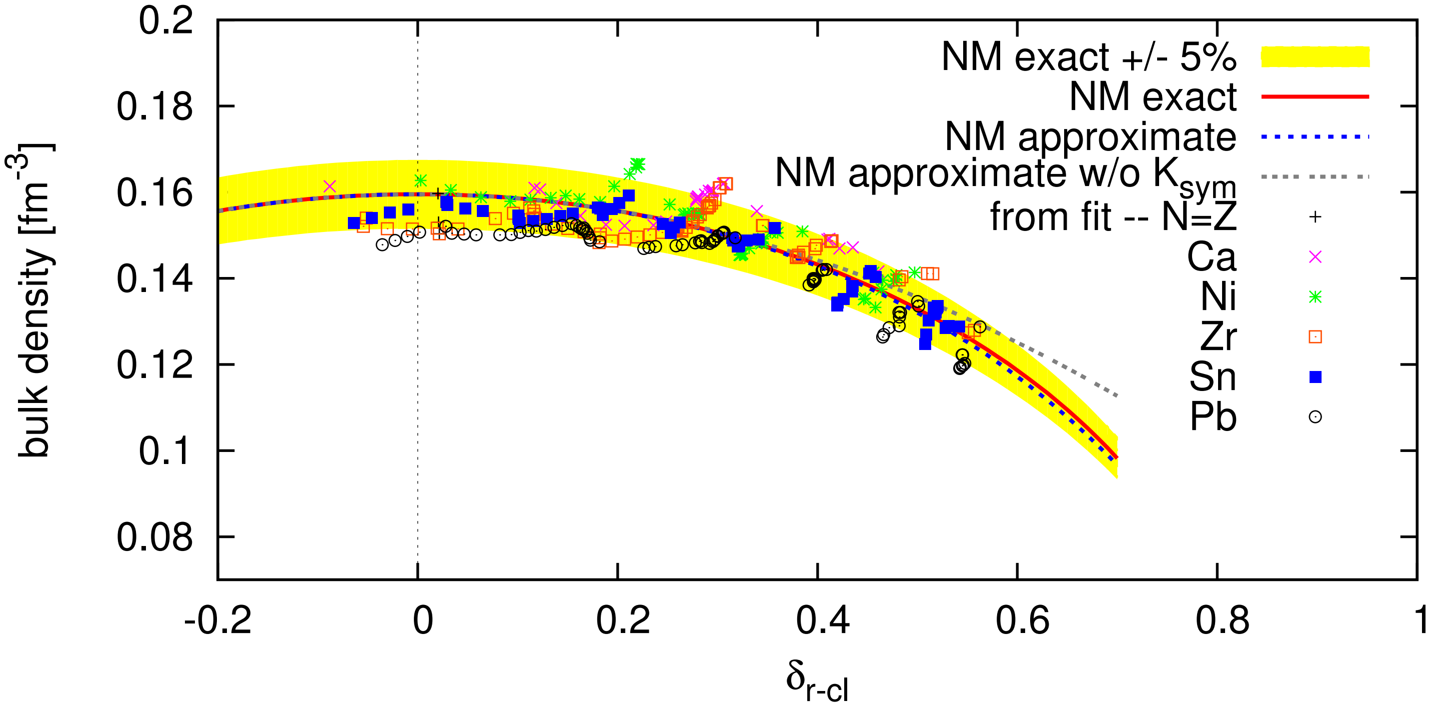




A simple model for the GS density

Hypothesis 2: the density at the center of the cluster is that of a uniform gas at mechanical stability.

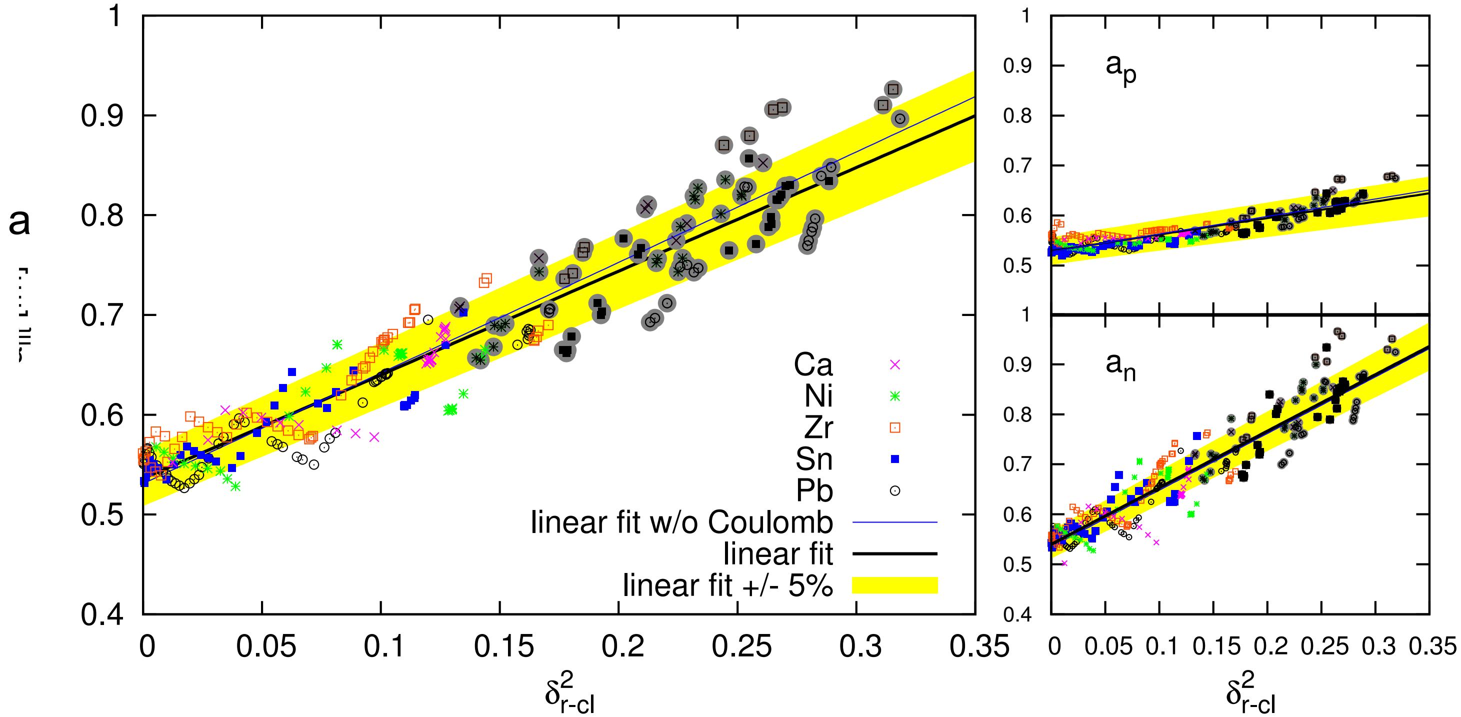
$$\rho_0^{GLDM,n=2}(\delta_{r-cl}) = \varrho_0 \left(1 - \frac{3L\delta_{r-cl}^2}{K_\infty + K_{sym}\delta_{r-cl}^2}\right)$$



Papakonstantinou et al., PRC 2013

A simple model for the GS density

Hypothesis 3: the diffuseness *a* depends only of the asymmetry of the cluster and not the mass.



Papakonstantinou et al., PRC 2013

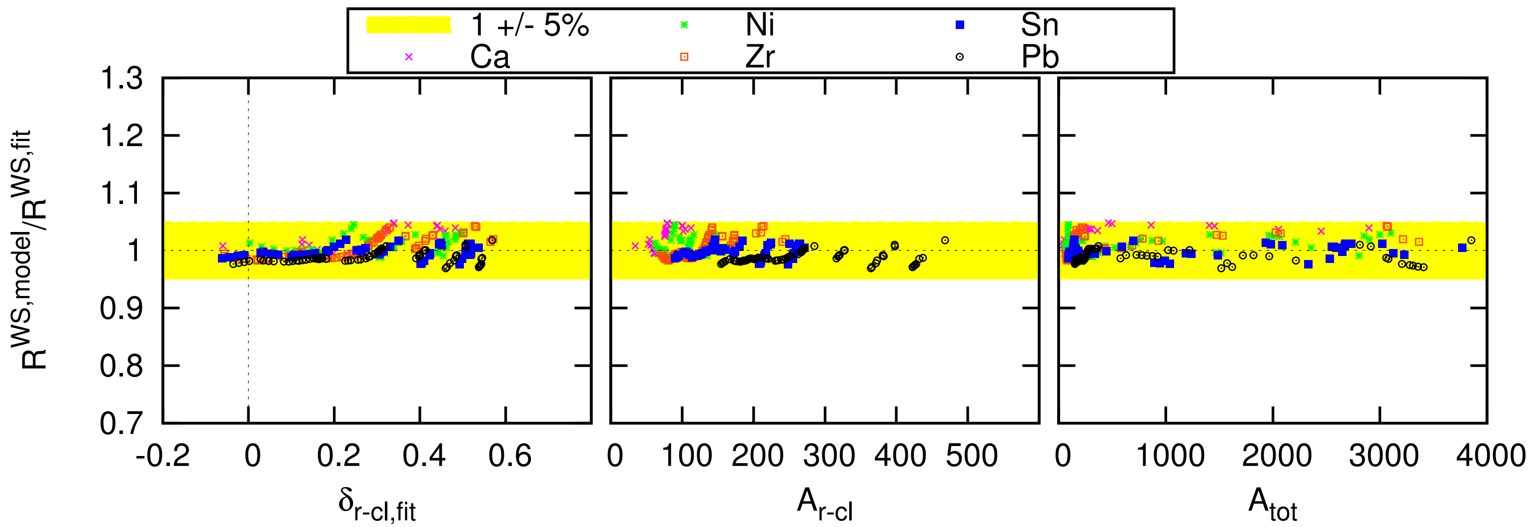
A simple model for the GS density

Hypothesis 4: The cluster radius is that of a uniform density sphere.

$$V^{HS} = \frac{A_{r-cl}}{\rho_0(\delta_{r-cl})} \equiv V_{cl} \quad \Longrightarrow \quad R^{HS} = \left(\frac{3V^{HS}}{4\pi}\right)^{1/3}$$

The Wigner-Seitz parameter R^{WS}:

$$R^{WS} = R^{HS}\left[1 - \frac{\pi^2}{3}\left(\frac{a}{R^{HS}}\right)^2\right]$$



Papakonstantinou et al., PRC 2013

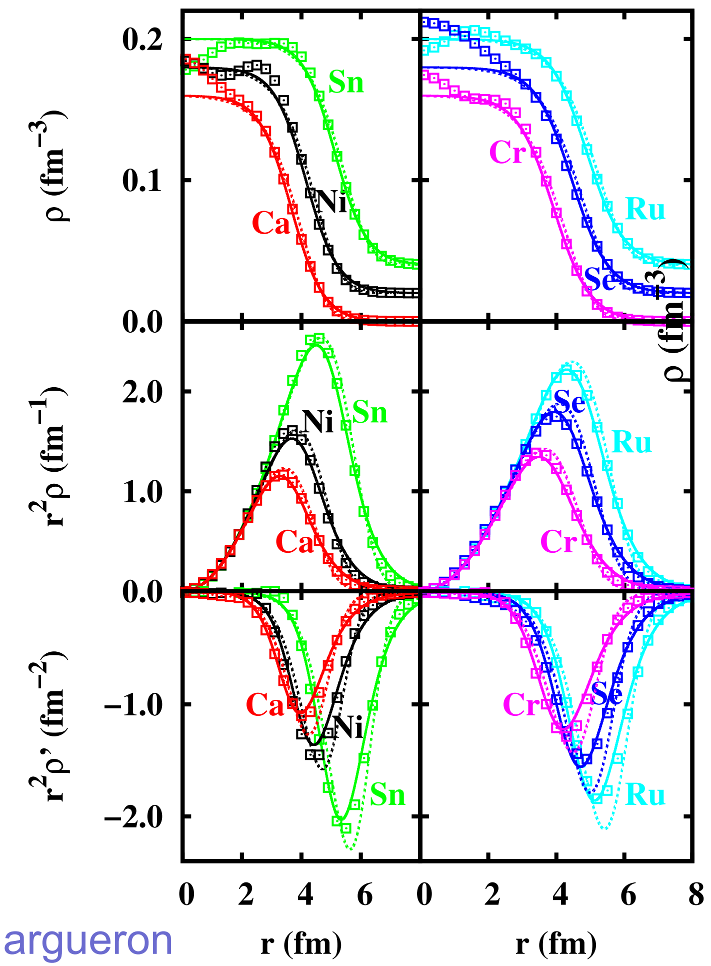
A simple model for the GS density

We solve the set of coupled equation and obtain the parameters of the WS density.

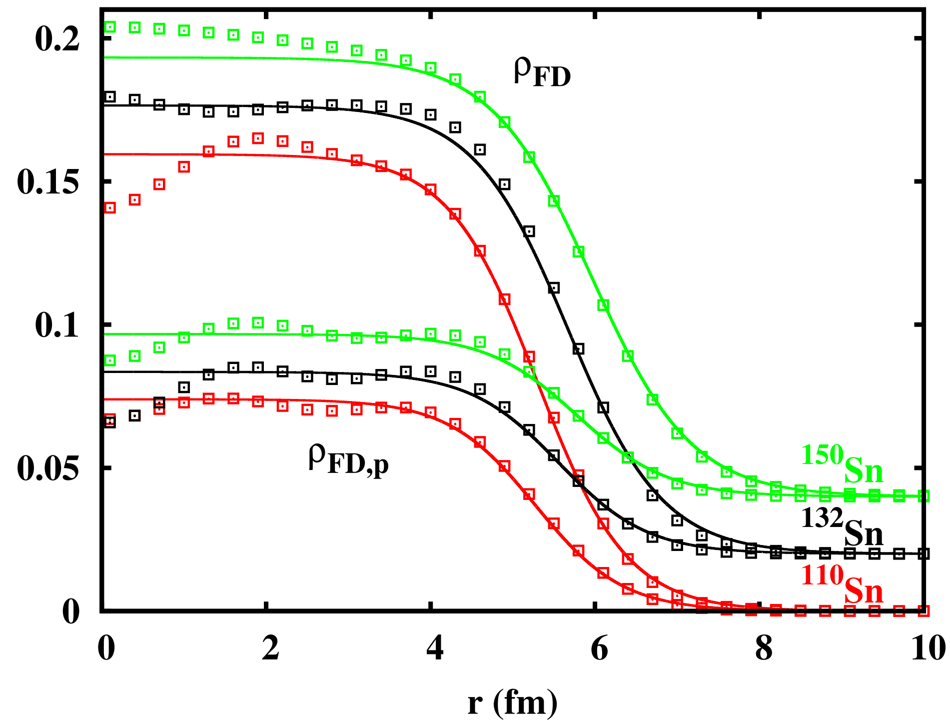
Very good agreement with HF GS quantities (5% accuracy on WS parameters).

Aymard et al. 2014

N=Z nuclei



Sn isotopes



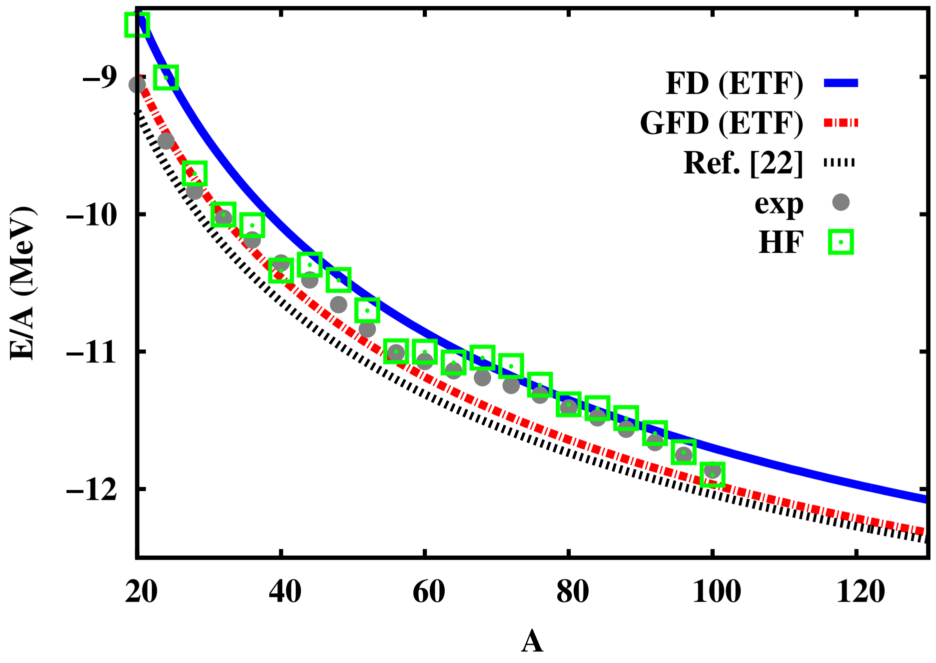
Close relation between the density profiles and some empirical coefficients

GS bulk and surface energies

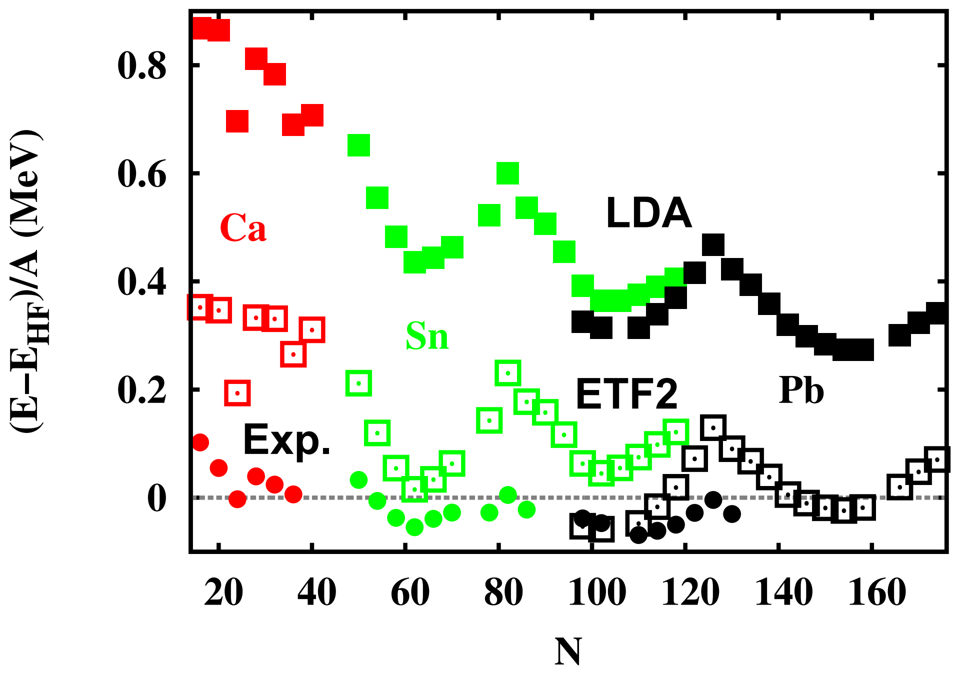
Nuclei energy

We use extended Thomas-Fermi model to link the density profiles to the total energy.

N=Z nuclei



Ca, Sn & Pb isotopes

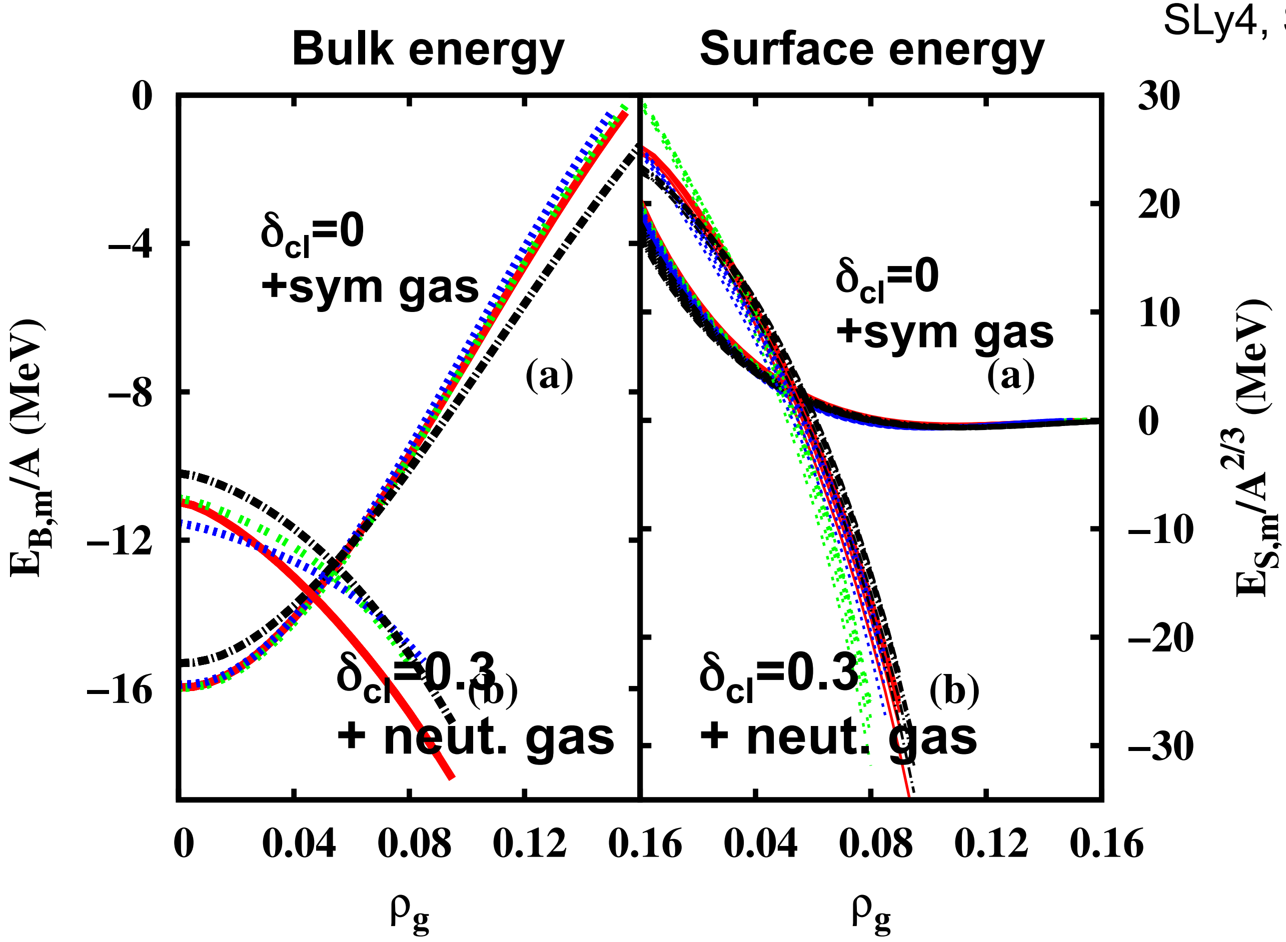


Aymard et al. 2014

Medium correction to cluster energies

The finite density of the gas changes the energy of the clusters.

Comparison of
SLy4, SKI3, LNS, SGI



Aymard et al. 2014

+ Discussion of the sign of the surface symmetry coefficient

Summary and outlook

- Simple model which reproduce the HF GS density profiles **in isolated nuclei** and **dilute nuclei**
- Use of **EFT** to deduce the total energy in the Wigner-Seitz cell.
  → *correction to LDM or experimental energies used in NS crust or SuperNovae.*

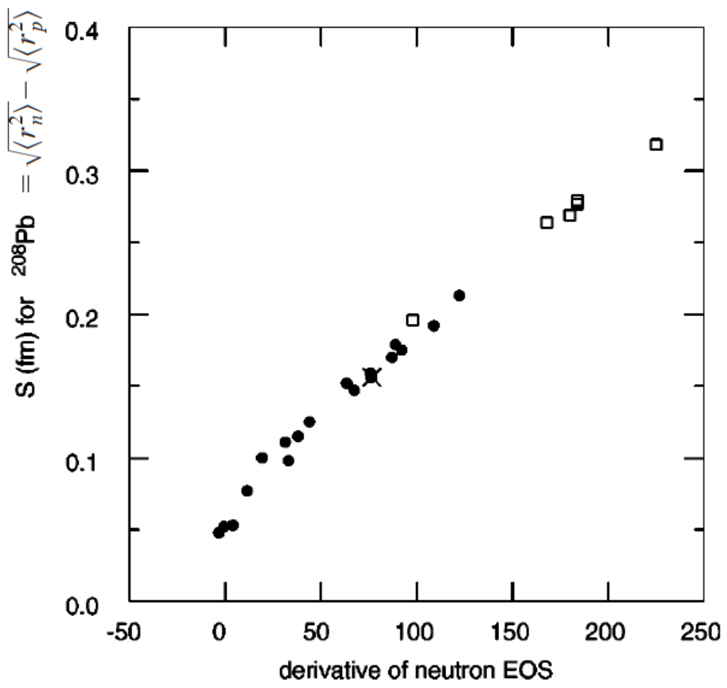
Relation between the density and energetic properties of clusters with the ***empirical coefficients****.*

Analytical expressions for the diffuseness parameter and for the gas contribution are underway…   Aymard et al., 2014 in preparation

Role of the empirical coefficient on the EoS

From Jorge's talk: the EoS is largely determined by L (**slope of the symmetry energy**)

Mixing of NR and R models



Typel & Brown, PRC 64, 027301 (2001)

→ **The radius of NS** is strongly related to **L**.

The EoS is not entirely reduced to L.

What is the effect of K_0, K_{sym}, …

A flexible parameterisation of the EoS

Requirements:

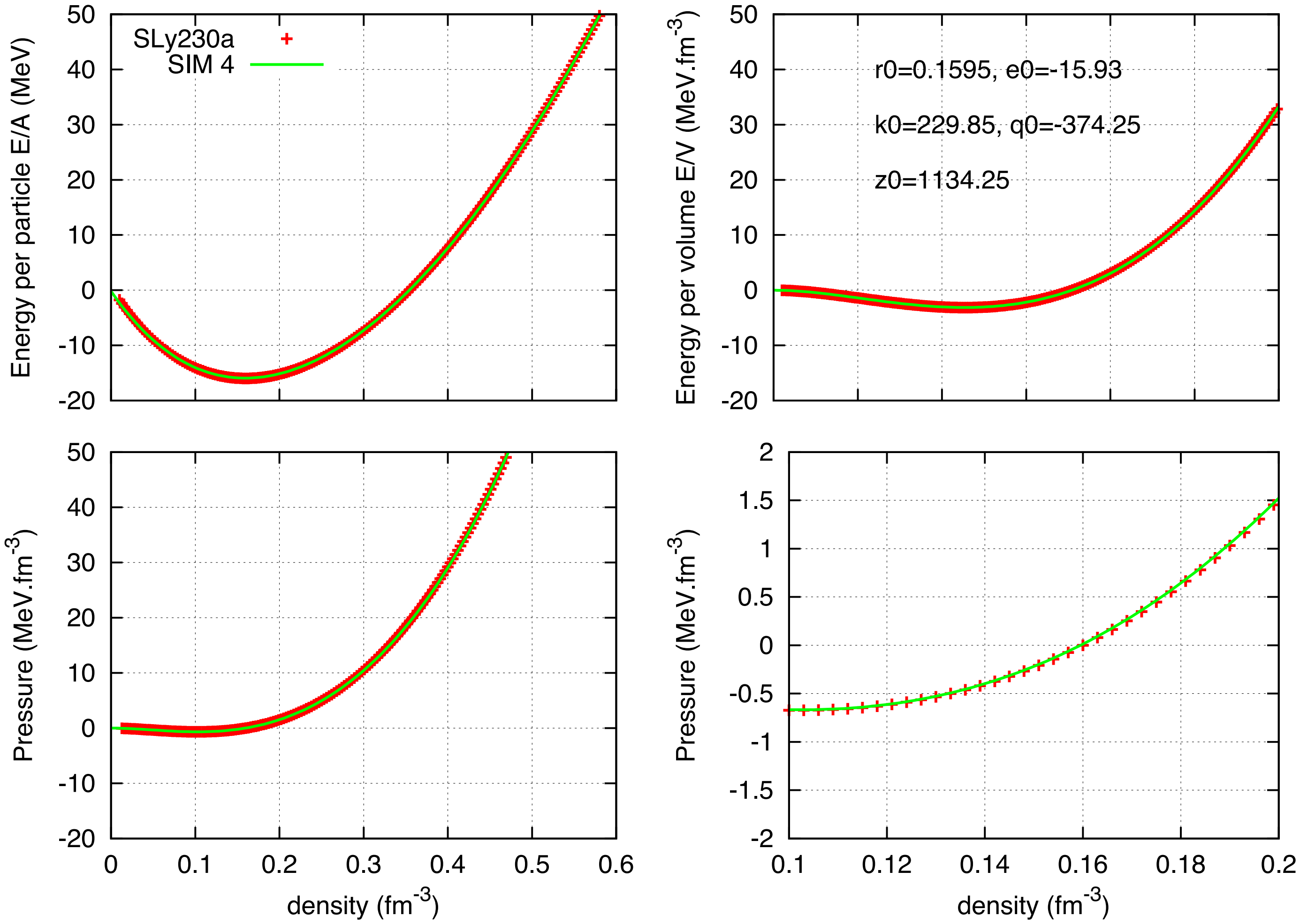
- ✧ The model shall be as **flexible** as possible, eventually at the price of increasing the number of parameters.
- ✧ We want to control at best the **density dependence** of the EoS, and of all its **derivatives**. We want to be able to fix all the derivatives, but one, in a simple way.
- ✧ The model shall include an estimation on the **theoretical error bars** in the **extrapolation** to unknown regions.
- ✧ The relation between **experimental constraints** and the **parameters of the model** shall be simple/direct and clear.

How:

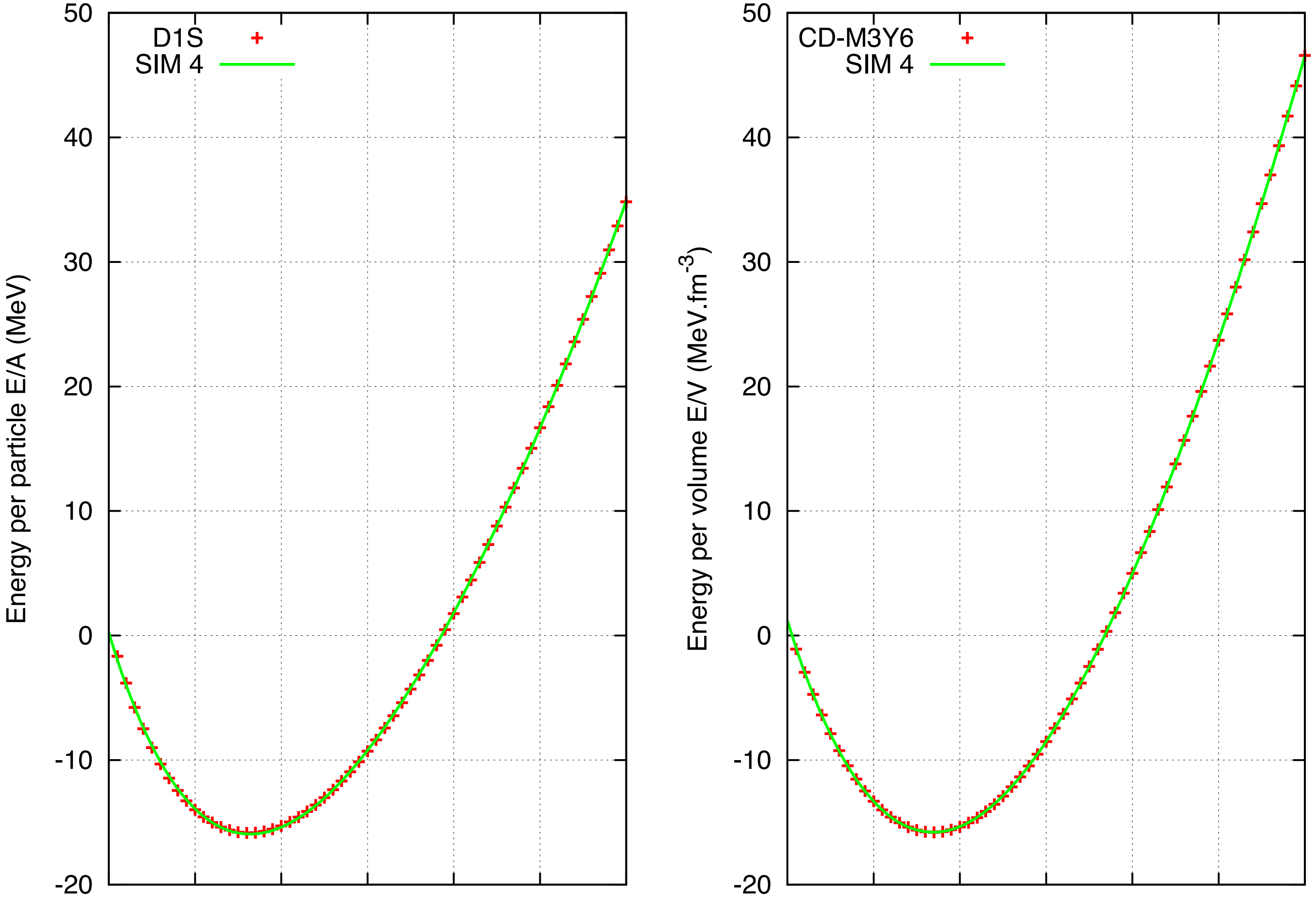
- ✧ We take advantage of **the density functional theory** → $E(\rho,\delta)$.
- ✧ We take a **reference density**, for instance the saturation density in symmetric matter → ρ_0.
- ✧ We decompose the energy into: a kinetic energy + potential (non-relativistic model).
- ✧ The parameters of the model are the *n*-**derivative** of the EoS at ρ_0.

How good does it works?

Can SI model reproduces the Skyrme SLy230a EoS ?



Can SI model reproduces D1S (Gogny) & CD-M3Y6 ?



Parameters adjustment: experimental knowledge and uncertainties

In theory:

1- Adjust the first derivatives on nuclei properties (energy, radii, collective modes, …)
2- Use NS and SN constraints for higher order derivatives.

Short-cut:

We consider a set of nuclear models & calculate the empirical properties: E_0, K_0, E_{sym}, L_{sym}, K_{sym}, …, (=the n-derivatives of the EoS).

We assume that the experimental constraints on nuclei should give approximately the mean value of these parameters ± standard deviation.

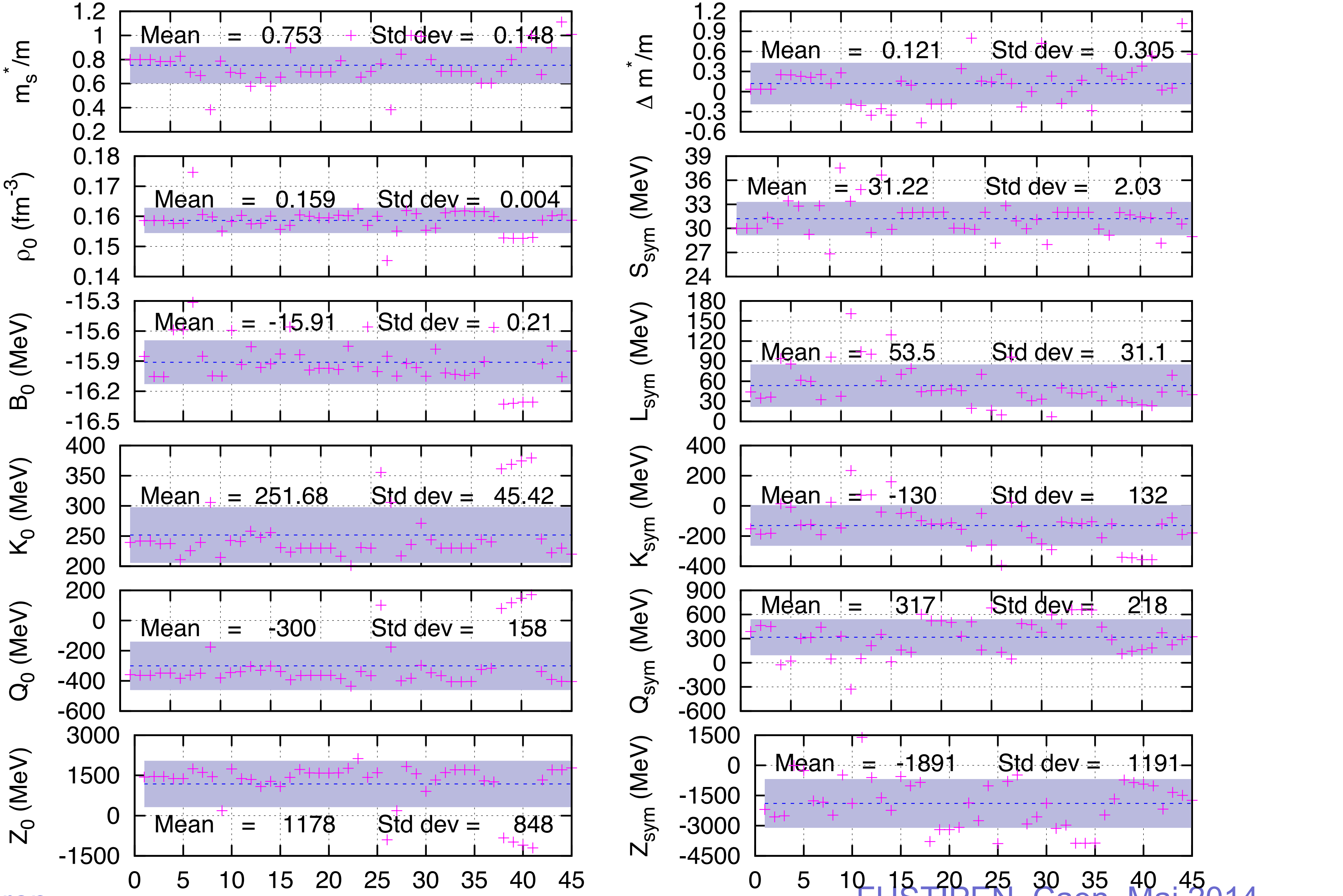
Reference model

The set of "*experimental*" empirical quantities represents our knowledge and uncertainties on the properties of the EoS.

Considering 45 Skyrme models:

	ρ_0	E_0	K_0	Q_0	Z_0	S_{sym}	L_{sym}	K_{sym}	Q_{sym}	Z_{sym}	M^*_s/M	$\Delta M^*/M$	$\bar{M}$	$\bar{\Delta}$
	fm^{-3}	MeV	MeV	MeV	MeV	MeV	MeV	MeV	MeV	MeV				
Average	0.1586	-15.91	251.68	-300.20	1178.35	31.22	53.52	-130.15	316.68	-1890.99	0.75	0.12	0.39	-0.07
χ	0.0040	0.21	45.42	157.81	848.47	2.03	31.06	132.03	218.23	1191.23	0.15	0.30	0.33	0.24
Max	0.1746	-15.31	379.40	171.65	2127.56	37.51	160.95	234.47	682.41	1391.17	1.11	1.02	1.61	0.49
Min	0.1453	-16.33	201.03	-435.56	-1201.67	26.83	7.02	-393.90	-327.67	-3893.87	0.38	-0.47	-0.10	-0.44

The "*experimental*" empirical quantities



Reference model

The set of "*experimental*" empirical quantities represents our knowledge and uncertainties on the properties of the EoS.

Model		ρ_0	E_0	K_0	Q_0	Z_0	E_{sym}	L_{sym}	K_{sym}	Q_{sym}	Z_{sym}
		fm^{-3}	MeV	MeV	MeV	MeV	MeV	MeV	MeV	MeV	MeV
Skyrme	Average	0.1586	-15.91	251.68	-300.20	1178.35	31.22	53.52	-130.15	316.68	-1890.99
	σ	0.0040	0.21	45.42	157.81	848.47	2.03	31.06	132.03	218.23	1191.23
RMF	Average	0.1494	-16.24	267.99	-1.94	5058.30	35.11	90.20	-4.58	271.07	-3671.83
	σ	0.0025	0.06	33.52	392.51	2294.07	2.63	29.56	87.66	357.13	1582.34
RHF	Average	0.1540	-15.97	248.06	389.17	5269.07	33.97	90.03	128.16	523.29	-9955.49
	σ	0.0035	0.08	11.63	350.44	838.41	1.37	11.06	51.11	236.80	4155.74
Average		0.1540	-16.04	255.91	29.01	3835.24	33.43	77.92	-2.19	370.34	-5172.77
σ		0.0051	0.20	34.39	424.59	2401.14	2.64	30.84	142.71	298.54	4362.35

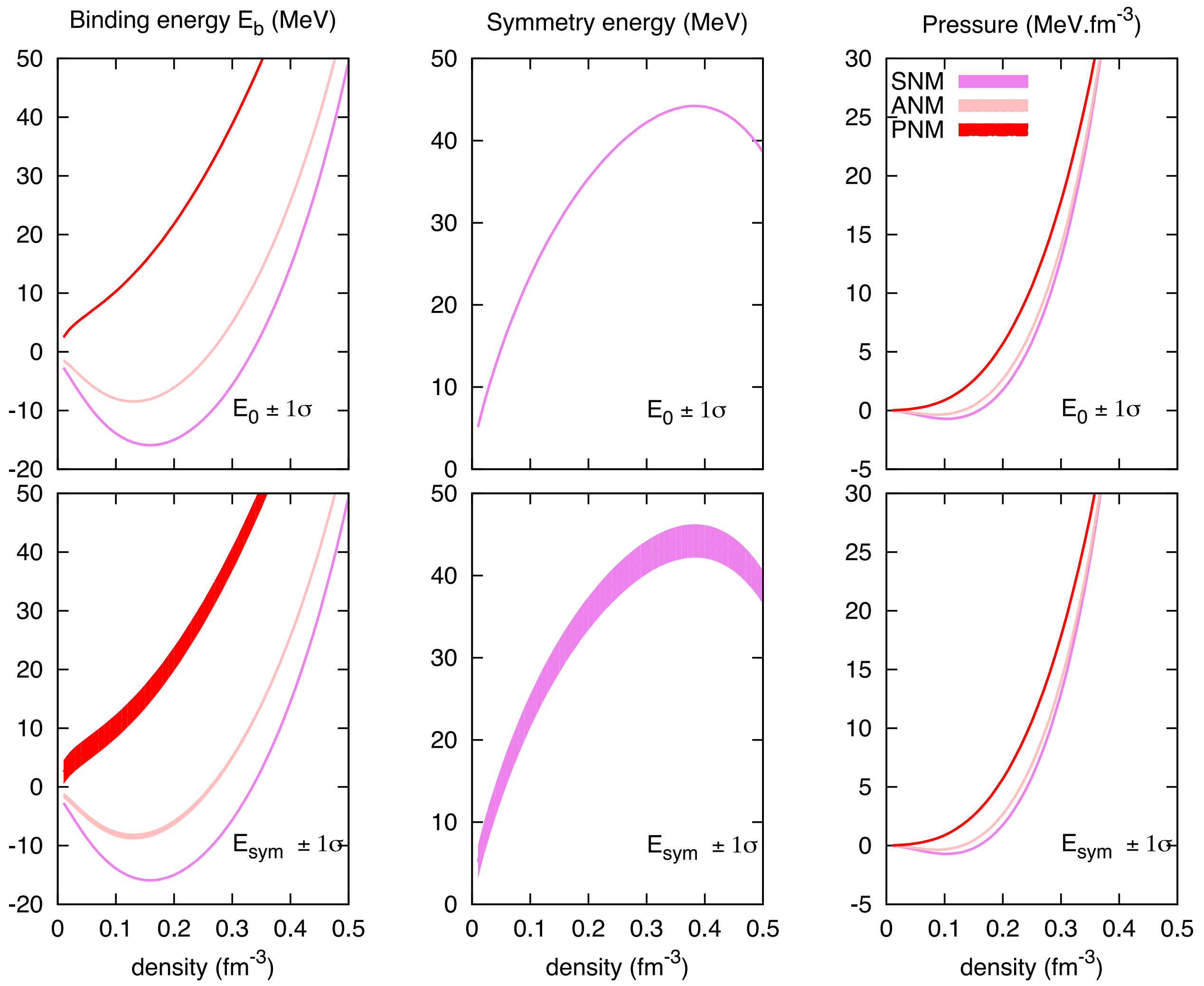
We vary our knowledge around the "experimental" average within ±1 σ.

We change 1 quantity, fixing all the others to the average value.

Impact of E_0 and E_{sym}

$$E_0 = E/A(\rho = \rho_0, \delta = 0)$$

$$E_{sym} = \frac{1}{2} \frac{\partial^2 E/A}{\partial \delta^2}|_{\rho=\rho_0, \delta=0}$$



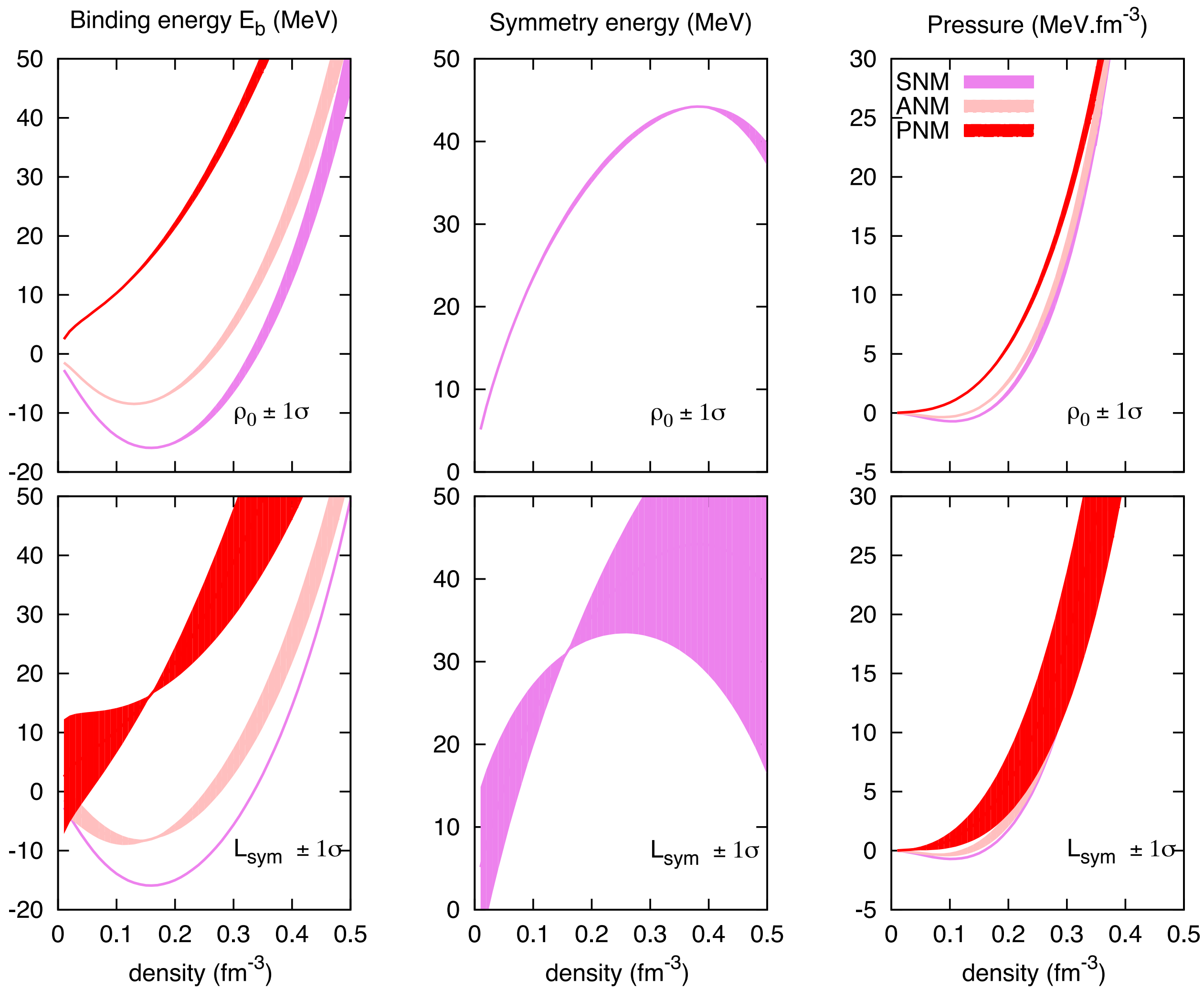
-15.91 ± 0.21

31.22 ± 2.03

Impact of ρ_0 & L_{sym} (1st derivatives)

$$P(\rho_0, \delta = 0) = 0$$

$$L_{sym} \# \frac{\partial E_{sym}}{\partial \partial \rho}|_{\rho=\rho_0}$$



0.1586

± 0.0040

53.52

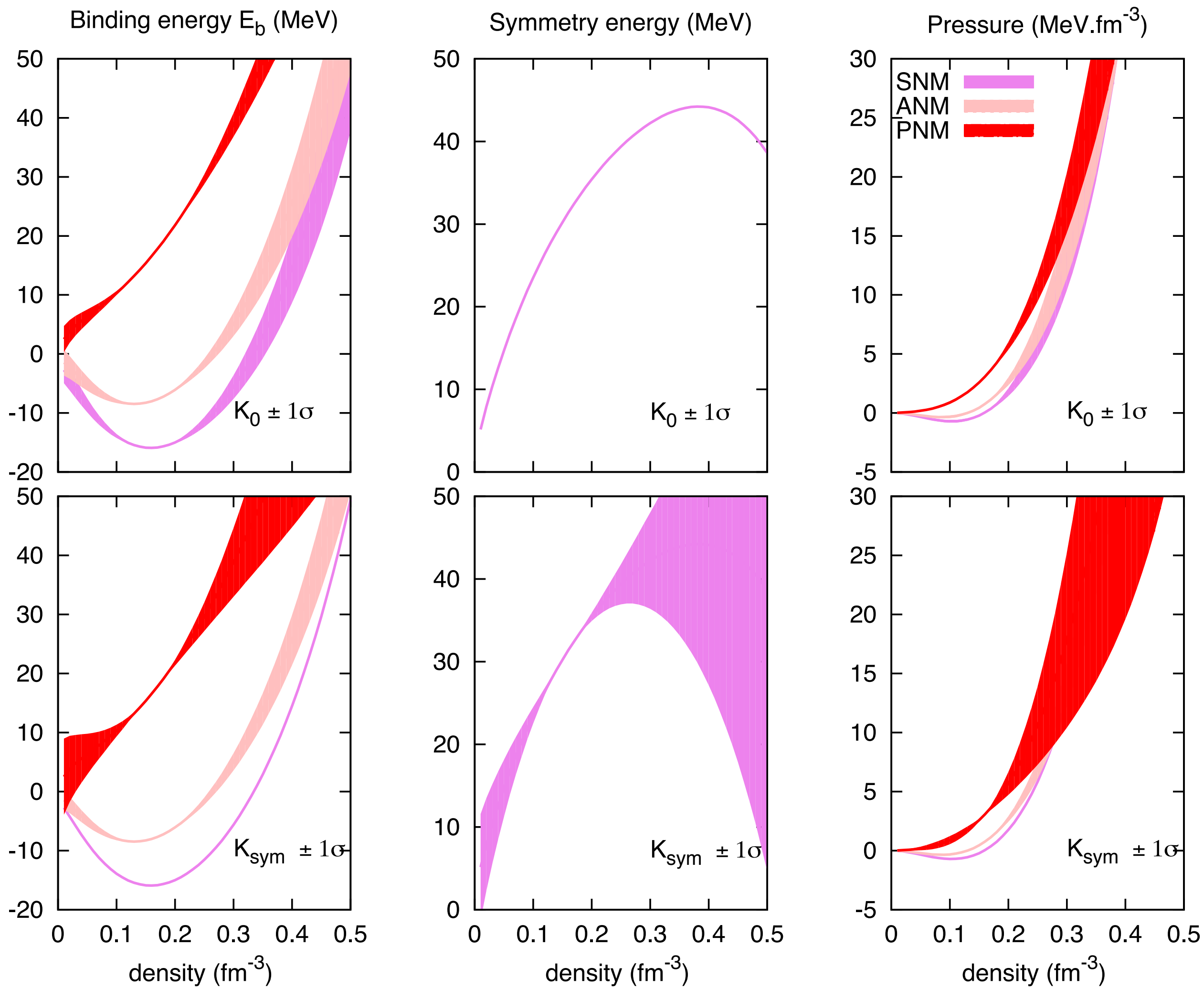
± 31.06

J. Margueron

Impact of K_0 & K_{sym} (2^{nd} derivatives)

$$K_0 \# \frac{\partial^2 E_0}{\partial \rho^2}|_{\rho_0}$$

$$K_{sym} \# \frac{\partial^2 E_{sym}}{\partial \rho^2}|_{\rho_0}$$



251.68

± 45.42

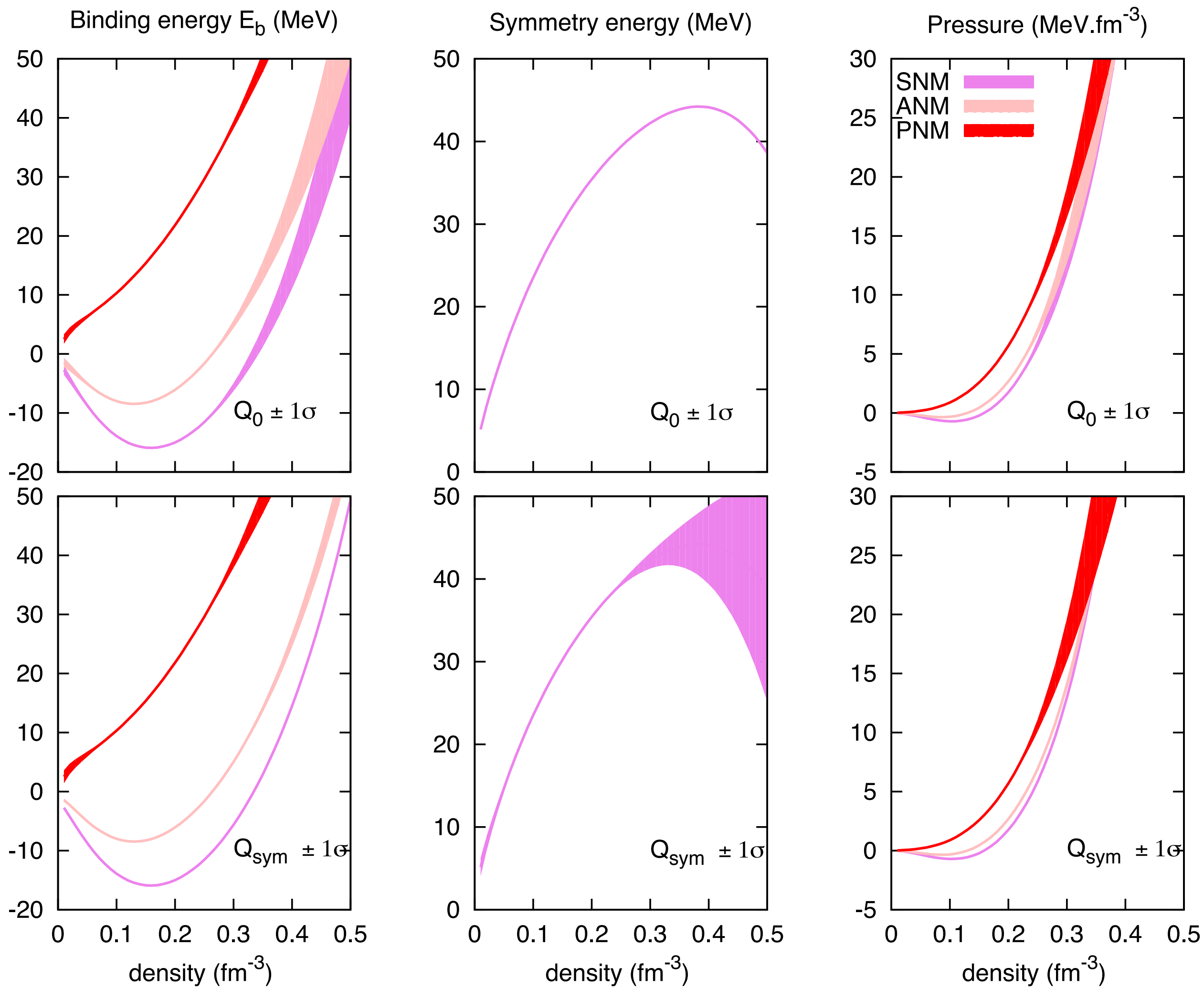
-130.15

± 132.03

Impact of Q_0 and Q_{sym} (3^{rd} derivatives)

$$Q_0 \# \frac{\partial^3 E_0}{\partial \rho^3}|_{\rho_0}$$

$$Q_{sym} \# \frac{\partial^3 E_{sym}}{\partial \rho^3}|_{\rho_0}$$



-300.20

± 157.81

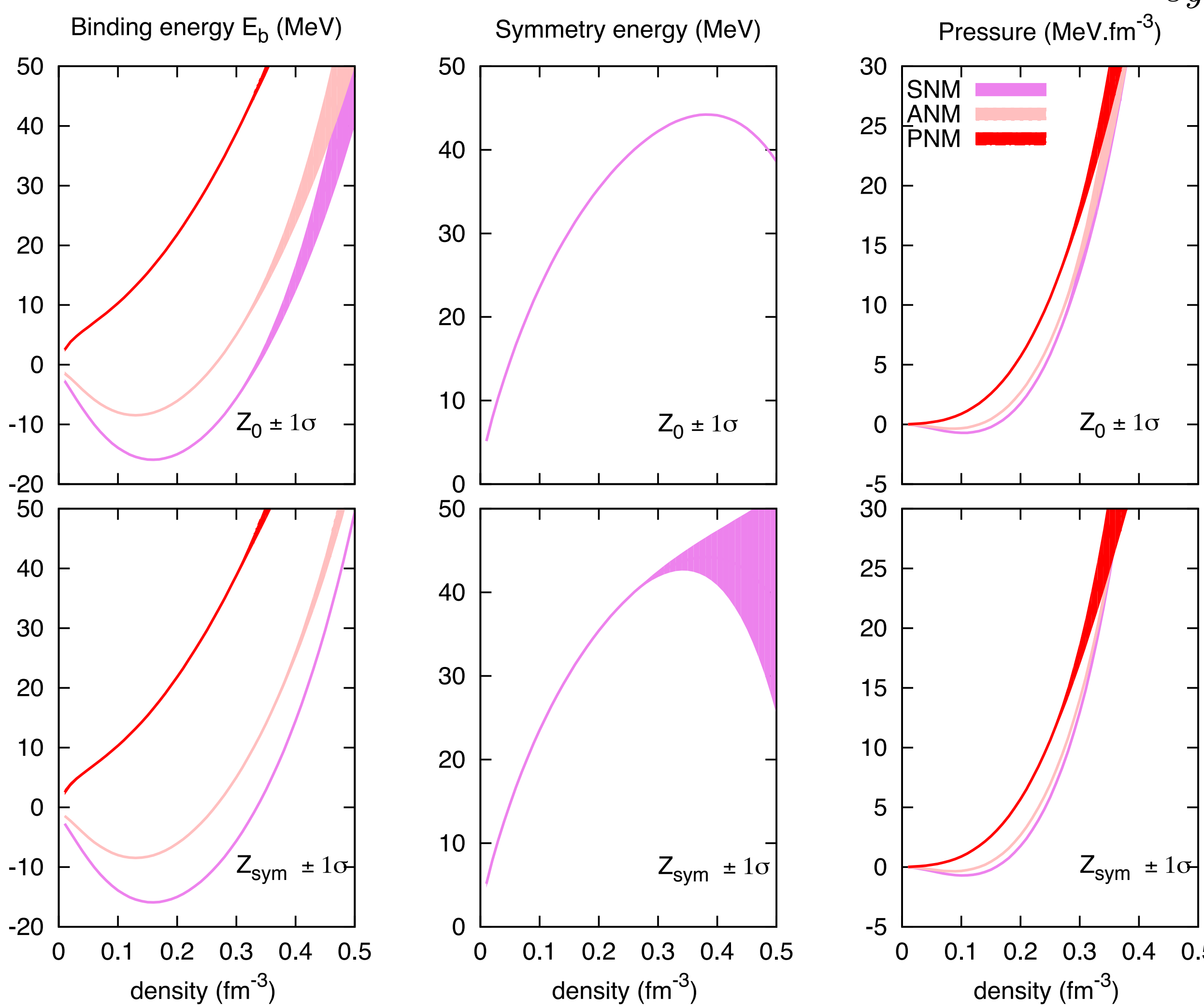
316.68

± 218.23

Impact of Z_0 and Z_{sym} (4^{th} derivatives)

$$Z_0 \# \frac{\partial^4 E_0}{\partial \rho^4}|_{\rho_0}$$

$$Z_{sym} \# \frac{\partial^4 E_{sym}}{\partial \rho^4}|_{\rho_0}$$



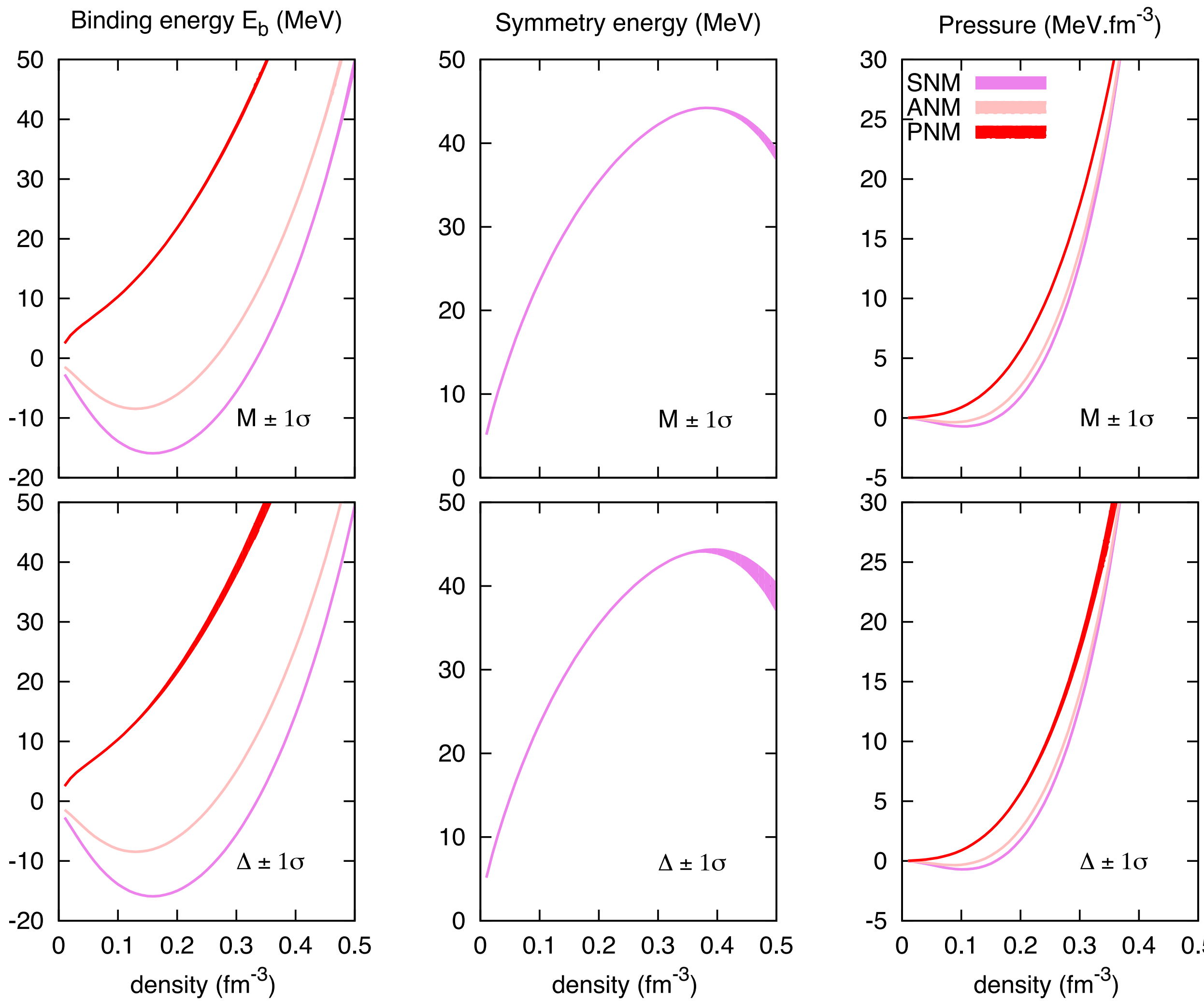
1178.35

± 848.47

-1890.99

± 1191.23

Impact of the in-medium effective mass



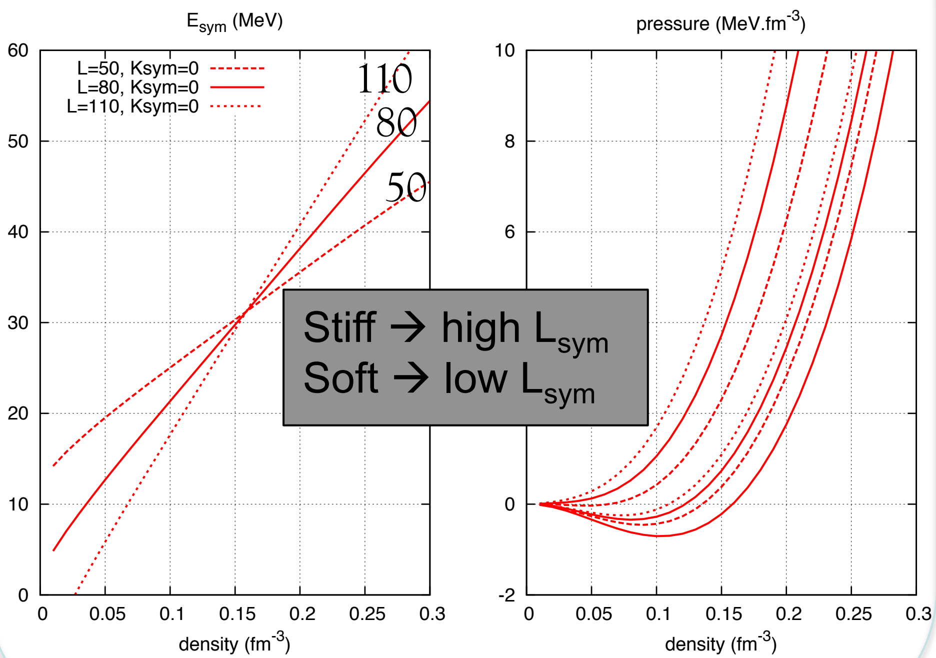
0.75 ± 0.15

0.12 ± 0.30

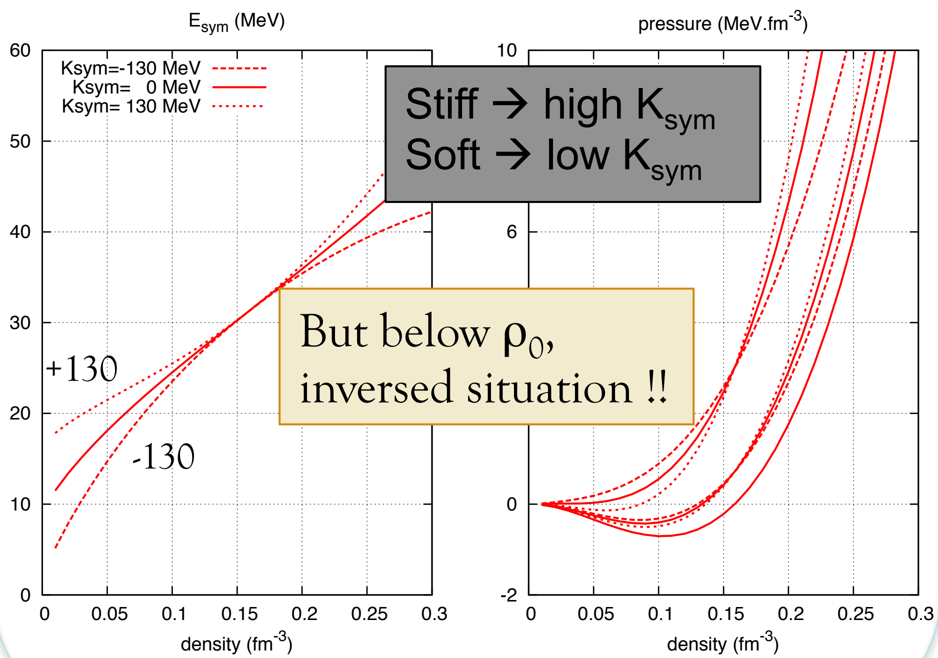
What means stiff / soft EoS ?

K_0 has a moderate impact on the EoS, compared to L_{sym} and K_{sym}.

Effect of L_{sym}



Effect of K_{sym}



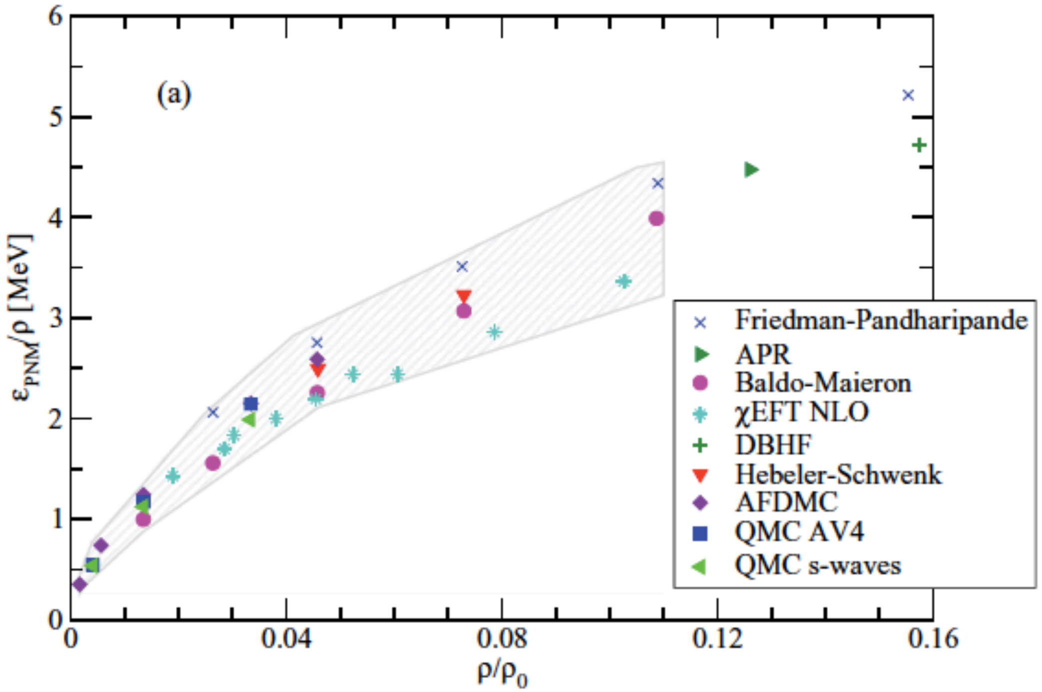
Reduction of the number of parameters

Include experimental, trustable theoretical and observational constraints at:

- sub-saturation density
- Supra-saturation density

Reduction of the number of parameters (low-density)

The low density neutron star EoS mostly depends of the low energy phase shift (S channel).
→ Convergence of theoretical predictions of the low density EoS:

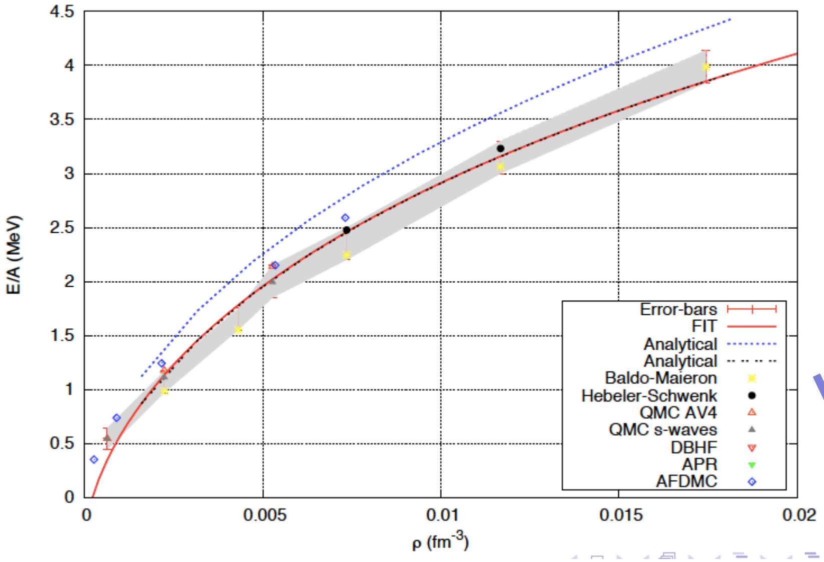


- On pure neutron matter.
- Fit with Baldo-Maieron, Hebeler-Schwenk, QMC AV 4 and QMC s-waves points.

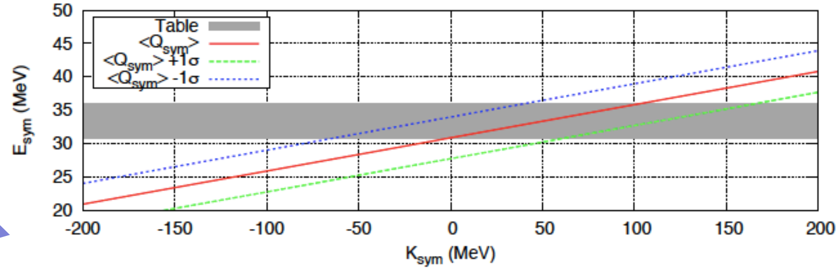
*Dutra et ai. PRC 85, 035201 (2012).

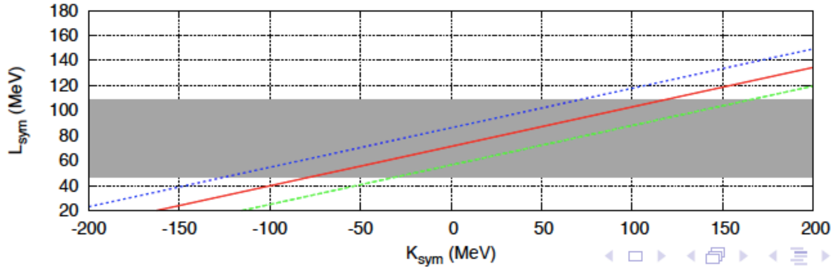
Reduction of the number of parameters (low density)

Preliminary results

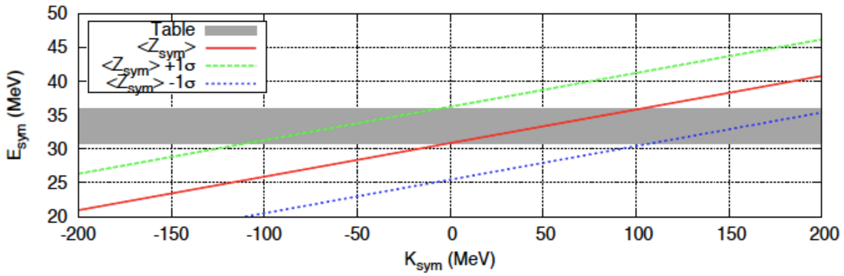


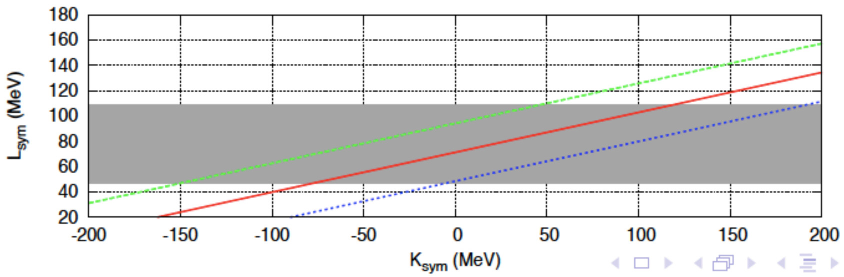
Fixed Q_{sym}





Fixed Z_{sym}



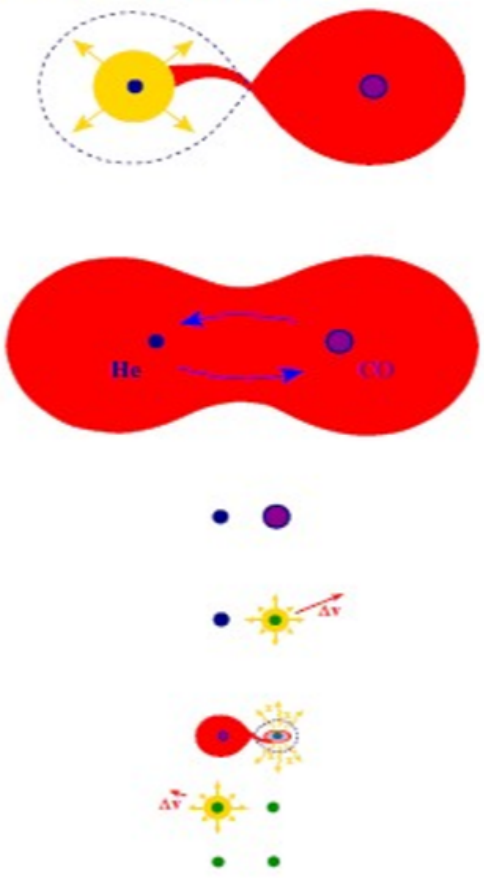


Reduction of the number of parameters (high density)

Courtesy of David Alvarez

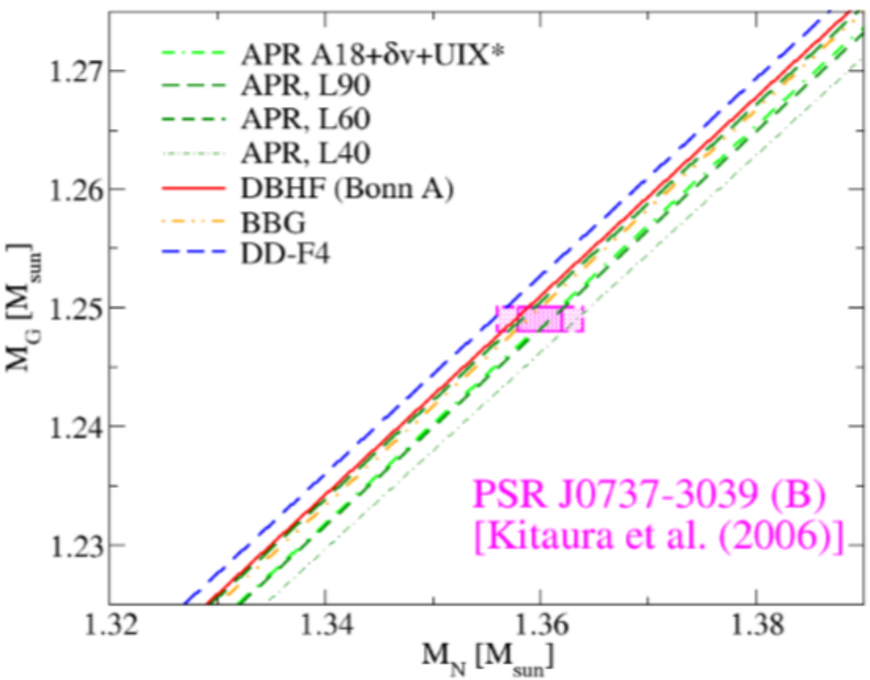
EoS constraint: double pulsar J0737-3039

Double core scenario:



Dewi et al., MNRAS (2006)

Baryon mass vs. gravitational mass - constraint or consistency check?



Podsiadlowski et al., MNRAS 361 (2005) 1243
Kitaura, Janka, Hillebrandt, A& A (2006); [astro-ph/0512065]

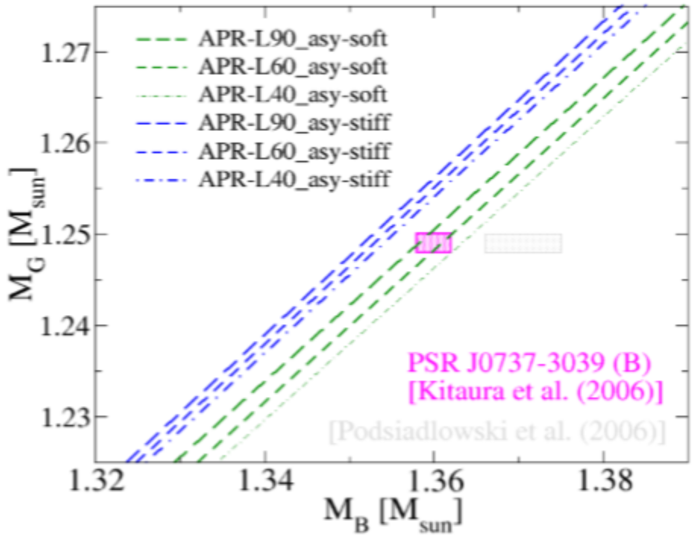
D.B., T. Klähn, F. Weber, CBM Physics Book (2008)

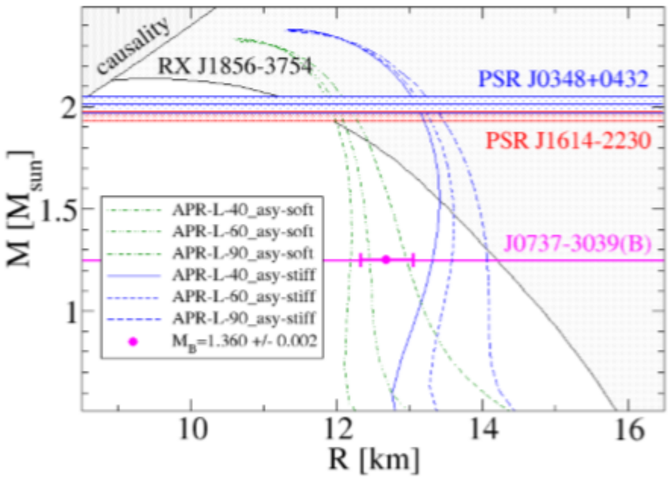
Reduction of the number of parameters (high density)

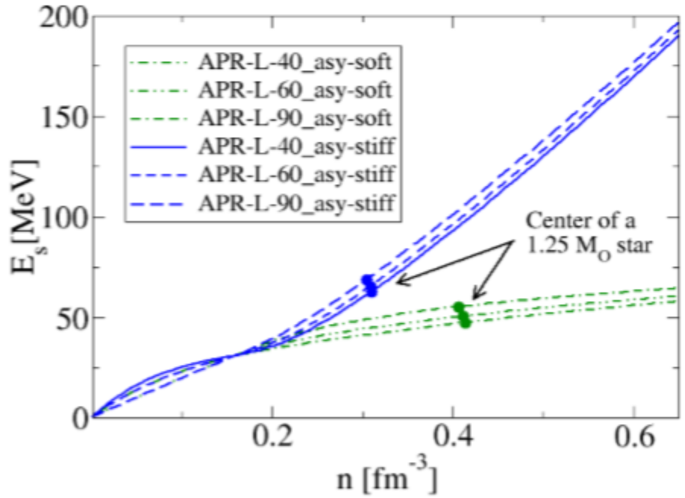
Courtesy of David Alvarez

Neutron star EoS: radius from M_B ?

- → M_B=1.36 M_O favors asy-soft EoS
- → ΔM_B= 0.002 M_O equiv to ΔL=20 MeV
- → lowering M_B at fixed M lowers the gravitational binding energy and increases the star radius







Conclusions and outlooks

- Going towards the drip line for 20<Z<40: role of resonant states and continuum coupling → consequences for inner/outer crust transition, finite temperature properties (reentrance phenomenon).
- The connexion NS physics and nuclear experiments can be performed through the correlation between the EoS and the empirical parameters.
- With a flexible paramerization of the EoS: the impact of the "*experimental*" uncertainty on our knowledge of the dense matter EoS can be accurately estimated. **L_{sym} and K_{sym} are very important parameters.**

Take home message:

The **link** between nuclear experiments and compact star physics is **rarely direct**.

The understanding of dense matter is **rarely** given by a **single experiment**, but better by the **accumulation of knowledge** in nuclear physics.

Neutron star **observations** can sometimes give **additional constraints** to nuclear models.

In collaboration with :

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Jiajie Li (IPN Lyon)
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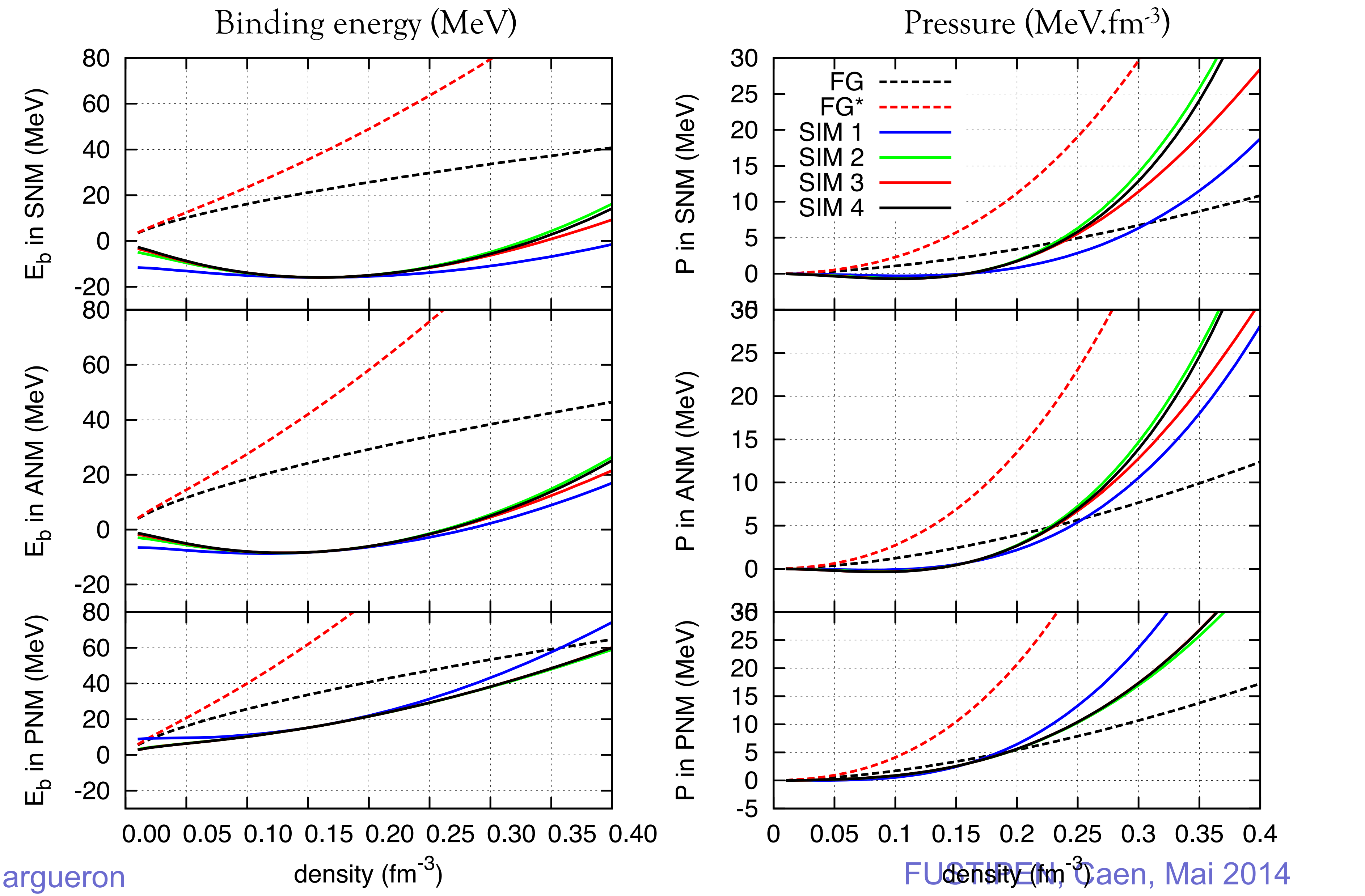
N. Sandulescu (NIPNE Bucharest),
P. Schuck (IPN Orsay),
X. Vinas (Univ. of Barcelona).

Effect of the different orders in the SI model

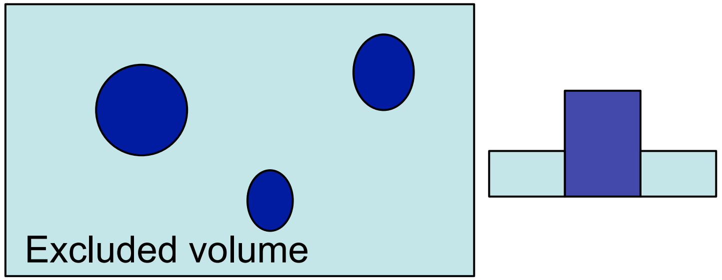


What is a dilute cluster?

In the semi-classical picture clusters are:
→ Piece of matter with density n ~ n_0 (as isolated nuclei),
→ Out of this cluster is located the dilute gas.

This picture is appropriate to employ LDM+excluded volume.

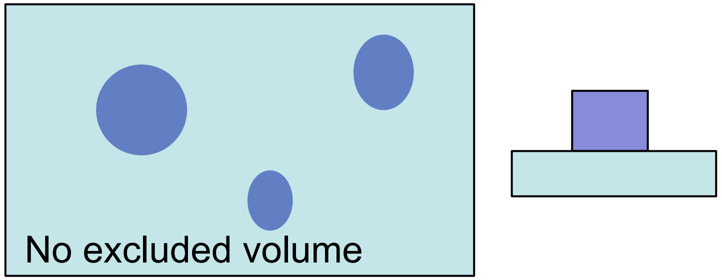
Coordinate-space clusters



But: in a quantum picture, nucleons from the gas can overlap the cluster:
→ Clusters are the bound states,
→ Gas is made of the continuum states.

In this picture, the LDM shall be corrected.

Energy-space clusters

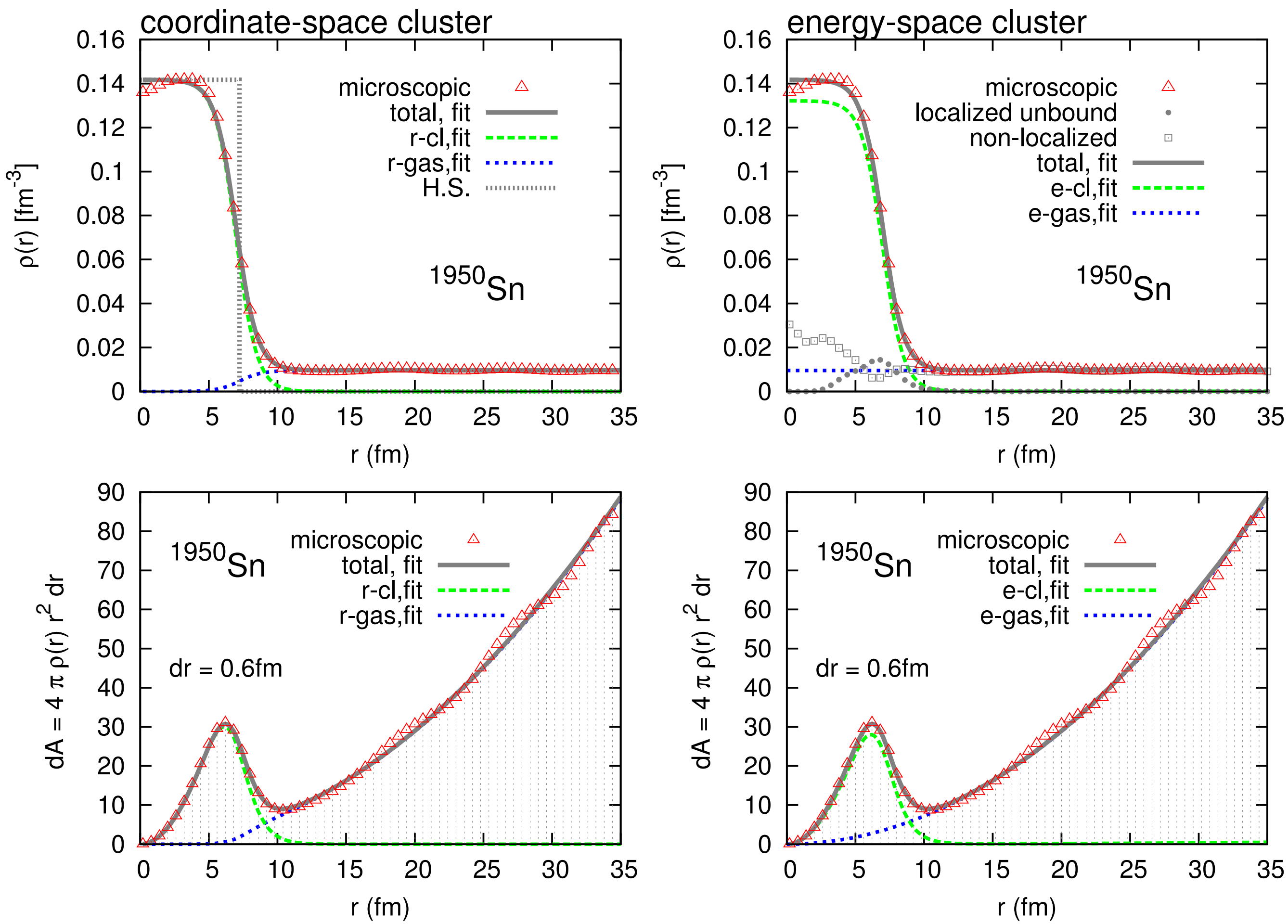


At low density, the semi-classical picture is not bad,
At high density, the overlapping betwen the gas and the cluster cannot be neglected.

However, these two pictures are just two simple representations of the same complex system.

Two representations of the same system:

HF model in a Wigner-Seitz cell



Papakonstantinou et al., PRC 2013

The densities are fitted to Wood-Saxon profiles:

$$\rho^{WS}_{r-cl,q}(r) = \frac{\rho_{0,q}}{1+\exp[(r-R^{WS}_q)/a^{WS}_q]}$$

$$\rho^{WS}_{r-gas,q}(r) = \frac{\rho_{gas,q}}{1+\exp[-(r-R^{WS}_q)/a_q]}$$

$$\rho^{WS}_{e-cl}(r) = \frac{\rho_0-\rho_{gas}}{1+\exp[(r-R^{WS})/a]}$$

$$\rho^{WS}_{e-gas}(r) = \rho_{gas}$$